

CSC304

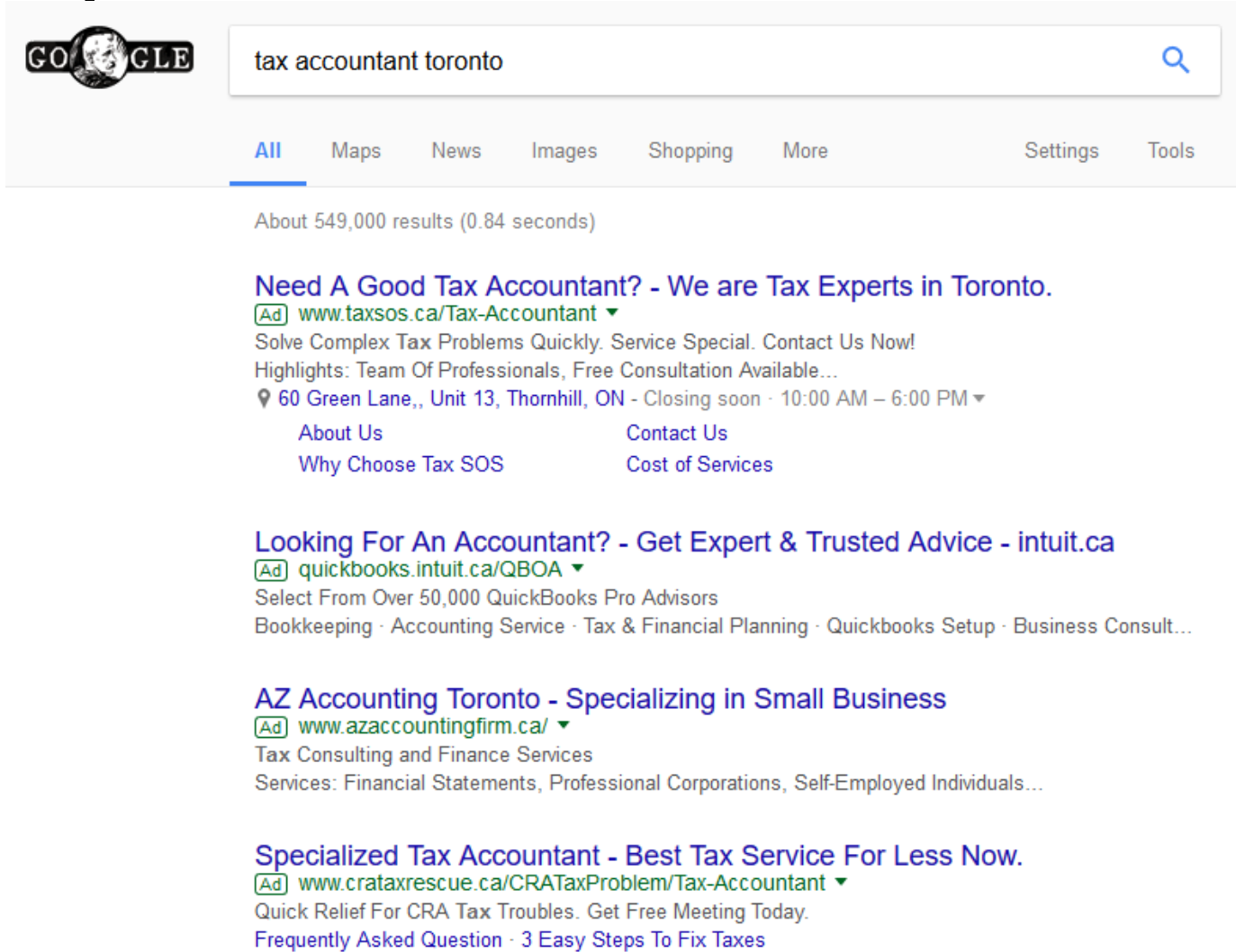
Algorithmic Game Theory & Mechanism Design

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This Lecture: More Auctions

- Sponsored search
- Other auction mechanisms
 - 1st price auction and ascending (English) auction
 - Comparison to the 2nd price auction
- A different type of incentive guarantee
 - Bayes-Nash Incentive Compatibility
- Revelation principle and revenue equivalence

Sponsored Search Auctions



The image shows a screenshot of a Google search results page. At the top left is the Google logo. The search bar contains the text "tax accountant toronto" and a magnifying glass icon. Below the search bar are navigation tabs: "All", "Maps", "News", "Images", "Shopping", "More", "Settings", and "Tools". The "All" tab is selected. Below the tabs, it says "About 549,000 results (0.84 seconds)". There are four sponsored search results listed, each with a blue title, a green "Ad" label, and a URL. The first result is for "www.taxesos.ca/Tax-Accountant" with a description about solving complex tax problems. The second is for "quickbooks.intuit.ca/QBOA" with a description about selecting from over 50,000 QuickBooks Pro advisors. The third is for "www.azaccountingfirm.ca/" with a description about tax consulting and finance services. The fourth is for "www.crataxrescue.ca/CRATaxProblem/Tax-Accountant" with a description about quick relief for CRA tax troubles.

GOOGLE

tax accountant toronto

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About 549,000 results (0.84 seconds)

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Quick Relief For CRA Tax Troubles. Get Free Meeting Today.
Frequently Asked Question · 3 Easy Steps To Fix Taxes

Sponsored Search Auctions

- A search engine receives a query
- There are k advertisement slots
 - “Clickthrough rates” : $c_1 \geq c_2 \geq \dots \geq c_k \geq c_{k+1} = 0$
- There are n advertisers (bidders)
 - Bidder i derives value v_i **per click**
 - Value to bidder i for slot $j = v_i \cdot c_j$
 - Without loss of generality, $v_1 \geq v_2 \geq \dots \geq v_n$
- **Question:**
 - Who gets which slot, and how much do they pay?



For convenience

Sponsored Search : VCG

- VCG

- Outcome

- Maximize welfare \Rightarrow bidder j gets slot j for $1 \leq j \leq k$, other bidders get nothing

- Payments

- Payment charged to bidder j = increase in welfare of others if j abstains

- Bidders $j + 1$ through $k + 1$ would be upgraded by one slot

- So:

- Payment of bidder $j = \sum_{i=j+1}^{k+1} v_i \cdot (c_{i-1} - c_i)$

- Payment of bidder j ***per click*** = $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1} - c_i}{c_j}$

Sponsored Search : VCG

- What if all the clickthrough rates are same?
 - $c_1 = c_2 = \dots = c_k > c_{k+1} = 0$
 - Payment of bidder j per click
 - $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1} - c_i}{c_j} = v_{k+1}$
 - Bidders 1 through k pay the value of bidder $k + 1$
 - Familiar? VCG for k identical items

Sponsored Search : GSP

- **Generalized Second Price Auction (GSP)**
 - For $1 \leq j \leq k$, bidder j gets slot j and pays the value of bidder $j + 1$ per click
 - Other bidders get nothing and pay nothing
- **Natural extension of the “second price” idea**
 - We considered this before for two identical slots
 - Not strategyproof
 - In fact, truth-telling may not even be a Nash equilibrium ☹️

Sponsored Search : GSP

NOT IN
SYLLABUS

- But there is a **good Nash equilibrium** that...
 - realizes the VCG outcome, i.e., **maximizes welfare**, and
 - generates **as much revenue as VCG** 😊 [Edelman et al. 2007]
- Even the **worst Nash equilibrium**...
 - gives **1.282-approximation to welfare** ($PoA \leq 1.282$) and
 - generates at least **half of the revenue of VCG**
[Caragiannis et al. 2011, Dutting et al. 2011, Lucier et al. 2012]
- So if the players achieve an equilibrium, things aren't so bad

VCG vs GSP

NOT IN
SYLLABUS

- VCG

- Truthful revelation is a dominant strategy, so there's a higher confidence that players will reveal truthfully and the theoretical welfare/revenue guarantees will hold
- But it is difficult to convey and understand

- GSP

- Need to rely on players reaching a Nash equilibrium
- But has good welfare and revenue guarantees and is easy to convey and understand

- Industry is split on this issue too!

From Theory to Reality

- **Value is proportional to clickthrough rate?**
 - Could it be that users clicking on the 2nd slot are more likely buyers than those clicking on the 1st slot?
- **Misaligned values of advertisers and ad engines?**
 - An advertiser having a high value for a slot does not necessarily mean their ad is appropriate for the slot
- **Market competition?**
 - What if there are other ad engines deploying other mechanisms and advertisers are strategic about which ad engines to participate in?

Bayes-Nash Incentive Compatibility

Bayesian Framework

- Useful for providing weaker incentive guarantees than strategyproofness
- **Strategyproofness:**
 - “It’s best for me to tell the truth even if I know what other players are doing, and regardless of what they are doing.”
- **Weaker guarantee:**
 - “I don’t *exactly* know what others are going to do, but I have some idea. In expectation, it’s best for me to tell the truth.”
 - Incomplete information setting

Bayesian Framework

- Each agent i 's valuation v_i is sampled from a distribution D_i
 - v_i 's are independent of each other
 - T_i = valuation space of agent i (support of $D_i \subseteq T_i$)
 - A_i = bid space of agent i
 - Agent i 's strategy $s_i: T_i \rightarrow A_i$ converts her valuation to her bid
- All agents know all D_i -s and all s_i -s, but only their own v_i
 - Agent i reasons about agent j 's bid in expectation over v_j drawn from D_j and then s_j applied to it

Bayesian Framework

- Given a **strategy profile** $\vec{s} = (s_1, \dots, s_n)$:

- **Expected utility** to agent i is

$$E_{\{v_j \sim D_j\}_{j \neq i}} [u_i(s_1(v_1), \dots, s_n(v_n))]$$

where utility u_i is “value derived – payment charged” under the outcome implemented when each agent j bids $s_j(v_j)$

- \vec{s} is a **Bayes-Nash equilibrium (BNE)** if s_i is the best strategy for agent i given \vec{s}_{-i} (strategies of others)
 - “I don’t know what others’ values are. But I know they are rational players, so I can reason about what strategies they might use.”

Comparison

- **Nash equilibrium**
 - **Given** their strategies and values, I'm doing the best I can
- **Bayes-Nash equilibrium**
 - **Given** their strategies and **in expectation over** their values, I'm doing the best I can
- **Dominant strategy equilibrium**
 - (Each player is playing their dominant action)
 - **Regardless of** their strategies and values, I'm doing the best I can

Example

- **Sealed-bid first price auction for a single item**
 - Each agent i confidentially submits a bid b_i
 - Agent i^* with the highest bid wins the item, pays b_{i^*}
- **Example**
 - Suppose there are two agents
 - Each agent i draws her valuation v_i for the item from the same distribution $U[0,1]$
 - **Claim:** Both players using the strategy $s(v) = v/2$ is a BNE.
 - Proof on the board.