CSC304
Algorithmic Game Theory & Mechanism Design

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This Lecture: More Auctions

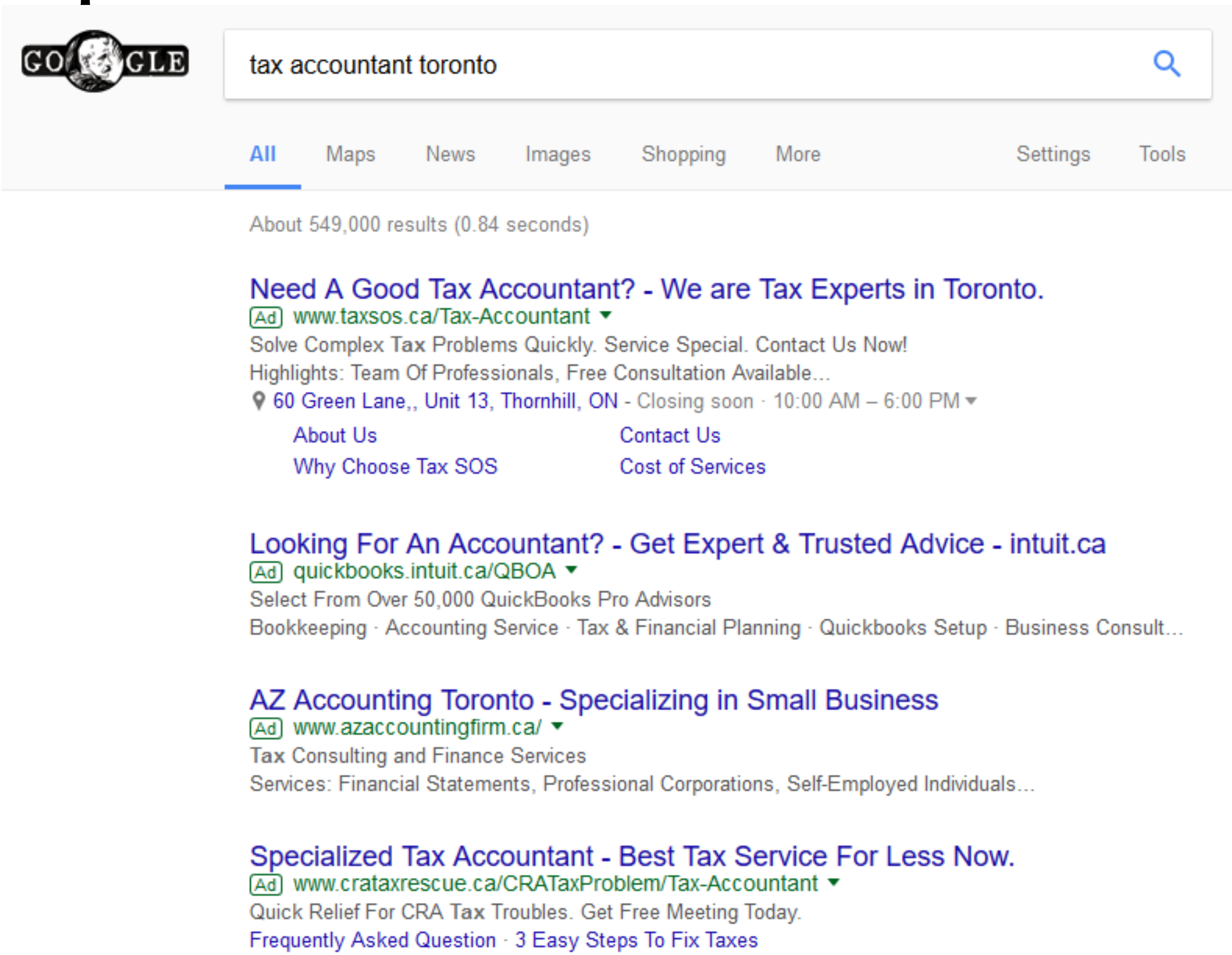
- Sponsored search

- Other auction mechanisms
  - 1^{st} price auction and ascending (English) auction
  - Comparison to the 2^{nd} price auction

- A different type of incentive guarantee
  - Bayes-Nash Incentive Compatibility

- Revelation principle and revenue equivalence

Sponsored Search Auctions



Sponsored Search Auctions

- A search engine receives a query
- There are k advertisement slots
  - "Clickthrough rates" : $c_1 \geq c_2 \geq \cdots \geq c_k \geq c_{k+1} = 0$

For convenience

- There are n advertisers (bidders)
  - Bidder i derives value v_i ***per click***
  - Value to bidder i for slot j = $v_i \cdot c_j$
  - Without loss of generality, $v_1 \geq v_2 \geq \cdots \geq v_n$
- Question:
  - Who gets which slot, and how much do they pay?

Sponsored Search : VCG

- VCG
  - Outcome
    - Maximize welfare $\Rightarrow$ bidder j gets slot j for $1 \leq j \leq k$, other bidders get nothing
  - Payments
    - Payment charged to bidder j = increase in welfare of others if j abstains
      - Bidders $j+1$ through $k+1$ would be upgraded by one slot
    - So:
      - Payment of bidder j = $\sum_{i=j+1}^{k+1} v_i \cdot (c_{i-1} - c_i)$
      - Payment of bidder j ***per click*** = $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1}-c_i}{c_j}$

Sponsored Search : VCG

- What if all the clickthrough rates are same?
  - $c_1 = c_2 = \cdots = c_k > c_{k+1} = 0$
  - Payment of bidder j ***per click***
    - $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1} - c_i}{c_j} = v_{k+1}$
  - Bidders 1 through k pay the value of bidder $k + 1$
    - Familiar? VCG for k identical items

Sponsored Search : GSP

- Generalized Second Price Auction (GSP)
  - For $1 \leq j \leq k$, bidder j gets slot j and pays the value of bidder $j+1$ ***per click***
  - Other bidders get nothing and pay nothing

- Natural extension of the "second price" idea
  - We considered this before for two identical slots
  - Not strategyproof
  - In fact, truth-telling may not even be a Nash equilibrium ☹

Sponsored Search : GSP

NOT IN SYLLABUS

- But there is a good Nash equilibrium that…
  - realizes the VCG outcome, i.e., maximizes welfare, and
  - generates as much revenue as VCG ☺ [Edelman et al. 2007]

- Even the worst Nash equilibrium…
  - gives 1.282-approximation to welfare ($PoA \le 1.282$) and
  - generates at least half of the revenue of VCG
    [Caragiannis et al. 2011, Dutting et al. 2011, Lucier et al. 2012]

- So if the players achieve an equilibrium, things aren't so bad

VCG vs GSP

NOT IN SYLLABUS

- VCG
  - Truthful revelation is a dominant strategy, so there's a higher confidence that players will reveal truthfully and the theoretical welfare/revenue guarantees will hold
  - But it is difficult to convey and understand
- GSP
  - Need to rely on players reaching a Nash equilibrium
  - But has good welfare and revenue guarantees and is easy to convey and understand
- Industry is split on this issue too!

From Theory to Reality

- Value is proportional to clickthrough rate?
  - Could it be that users clicking on the 2nd slot are more likely buyers than those clicking on the 1st slot?
- Misaligned values of advertisers and ad engines?
  - An advertiser having a high value for a slot does not necessarily mean their ad is appropriate for the slot
- Market competition?
  - What if there are other ad engines deploying other mechanisms and advertisers are strategic about which ad engines to participate in?

Bayes-Nash Incentive Compatibility

Bayesian Framework

- Useful for providing weaker incentive guarantees than strategyproofness

- Strategyproofness:
  - "It's best for me to tell the truth even if I know what other players are doing, and regardless of what they are doing."

- Weaker guarantee:
  - "I don't *exactly* know what others are going to do, but I have some idea. In expectation, it's best for me to tell the truth."
  - Incomplete information setting

Bayesian Framework

- Each agent i's valuation v_i is sampled from a distribution D_i
  - v_i's are independent of each other
  - T_i = valuation space of agent i (support of $D_i \subseteq T_i$)
  - A_i = bid space of agent i
  - Agent i's strategy $s_i: T_i \to A_i$ converts her valuation to her bid

- All agents know all D_i-s and all s_i-s, but only their own v_i
  - Agent i reasons about agent j's bid in expectation over v_j drawn from D_j and then s_j applied to it

Bayesian Framework

- Given a strategy profile $\vec{s} = (s_1, \dots, s_n)$:
  - Expected utility to agent i is

$$E_{\{v_j \sim D_j\}_{j \neq i}} \left[u_i\big(s_1(v_1), \dots, s_n(v_n)\big) \right]$$

    where utility u_i is "value derived – payment charged" under the outcome implemented when each agent j bids $s_j(v_j)$

  - $\vec{s}$ is a Bayes-Nash equilibrium (BNE) if s_i is the best strategy for agent i given $\vec{s}_{-i}$ (strategies of others)
    - "I don't know what others' values are. But I know they are rational players, so I can reason about what strategies they might use."

Comparison

- Nash equilibrium
  - Given their strategies and values, I’m doing the best I can

- Bayes-Nash equilibrium
  - Given their strategies and in expectation over their values, I’m doing the best I can

- Dominant strategy equilibrium
  - (Each player is playing their dominant action)
  - Regardless of their strategies and values, I’m doing the best I can

Example

- Sealed-bid first price auction for a single item
  - Each agent i confidentially submits a bid b_i
  - Agent i^* with the highest bid wins the item, pays b_{i^*}

- Example
  - Suppose there are two agents
  - Each agent i draws her valuation v_i for the item from the same distribution $U[0,1]$
  - Claim: Both players using the strategy $s(v) = v/2$ is a BNE.
    - Proof on the board.