CSC336 Term Test Summer 2002

1. (a) [5] For each of the following statements use $\top$ if you think the given statement is TRUE and $\perp$ if you think the given statement is FALSE. One mark is deducted for each wrong answer.

   i. Floating-point numbers are distributed uniformly throughout their range.

   ii. To obtain a more accurate summation of positive numbers in MATLAB, it is better to add the numbers from largest to smallest.

   iii. If A is any $n \times n$ nonsingular matrix, then $\kappa(\lambda A) = \kappa(A)$, where $\lambda \neq 0$.

   iv. If the residual $r = b - A\hat{x}$ of the computed solution $\hat{x}$ for the linear problem $Ax = b$ is small, then $\hat{x}$ is an accurate solution.

   v. When Newton's method converges, it always converges more quickly than the bisection method.

   (b) [5] Solve the following problem using least squares.

$$\begin{array}{ll} \text{minimize} & 9x_1^2 + 16x_2^2 \\ \text{subject to} & x_1 + 2x_2 = 3. \end{array}$$

2. The roots of $ax^2 + bx + c = 0$, when $a \neq 0$, are

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

   (a) [4] Using four-digit rounding arithmetic, apply this formula to the equation $x^2 + 62.10x + 1 = 0$, whose roots are approximately

$$x_1 = -0.01610723 \text{ and } x_2 = -62.08390.$$

   (b) [2] Explain why one of the computed roots is less accurate than the other.

   (c) [4] Suppose that one root is accurately computed. How would you use this fact to compute the other root? Use this method to improve the accuracy of one of the roots found in (a).

3. Consider the equation $Ax = b$ with

$$A = \begin{bmatrix} 6 & 5 & -10 \\ 8 & 4 & -12 \\ 6 & 4 & 7 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } b = \begin{bmatrix} -33 \\ -36 \\ 22 \end{bmatrix}$$

   (a) [6] Use Gaussian Elimination with row pivoting to compute the $PA = LU$ factorization of the coefficient matrix. Show P, L and U.

(b) [4] Use your $PA = LU$ factorization from part (a) to solve the problem $Ax = b$ for x.

4. (a) [5] Apply Newton's method to compute the reciprocal $1/a$ of a nonzero number a. Show that this can result in the iteration

$$x_{k+1} = x_k + x_k(1 - ax_k) \text{ for } k \geq 0.$$

(b) [5] Assume that this iterative method converges to the fixed point $1/a$. What is the order of convergence and the asymptotic error constant? Show your work.

Total marks = 40