CSC336 Term Test Summer 2001

1. (a) [5] For each of the following statements use $\top$ if you think the given statement is TRUE and $\perp$ if you think the given statement is FALSE. One mark is deducted for each wrong answer.

    i. Using higher-precision arithmetic will make an ill-conditioned problem better conditioned.

    ii. If a linear system $Ax = b$ is well-conditioned, then pivoting is unnecessary in GE.

    iii. If A is any $n \times n$ nonsingular matrix, then $\kappa(A) = \kappa(A^{-1})$.

    iv. When solving a nonsingular system of linear equations, GE with partial pivoting usually yields a solution with a small residual even if the matrix is ill-conditioned.

    v. If $f(x)$ is a real-valued function such that $f(1) < 0$ and $f(2) > 0$ then $f(x) = 0$ has a solution in the interval (1,2).

   (b) [5] Solve the system $Ax = b$ in the least square sense. Here

$$A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \\ 0 & 4 \end{bmatrix}, \; b = \begin{bmatrix} 6 \\ 13 \\ 4 \end{bmatrix}.$$

2. (a) [5] Derive a tight error bound for the computer evaluation of $x+y$. Assume that $x, y > 0$ but do not assume that x and y are exactly representable.

   (b) [5] Describe values for which you would expect the MATLAB function $f(x) = \sqrt{\frac{1}{x} + x} - \sqrt{\frac{1}{x} - x}$ to produce inaccurate results. Derive a more accurate formula that is mathematically equivalent to $f(x)$.

3. (a) [5] Find the LU factorization with partial pivoting of

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix}$$

   (b) [2] Find the matrix P such that $PA = LU$.

   (c) [3] Suppose you have the LU factorization as described in (b). Describe an effective method to solve the system $Ax = b$.

4. Consider the non-linear function $f(x) = \tan(x) - x$. It can be shown that $f'(x) = \tan^2(x)$. Recall that x is given in radians.

   (a) [2] Show that $f(x) = 0$ has a solution in the interval $[4.1, 4.7]$.

(b) [2] Would you recommend taking x_1 to be the result from applying one step of Newton's method to $f(x)$ with $x_0 = 4.1$? Justify.

(c) [1] What value would you recommend for x_1 if you require that x_1 be closer to the solution than x_0?

(d) [5] Compute an approximation x to the true solution α such that $|f(x)| \leq 1.0 \times 10^{-3}$ using Newton's method. You may use x_1 from (c) as your initial guess.