UNIVERSITY OF TORONTO
Faculty of Arts and Science

AUGUST EXAMINATIONS 2001

CSC 336H1 Y

Duration - 2 hours

Examination Aids: One 8.5×11 handwritten cheat sheet,
A non-programmable calculator.

1. [10]

   (a) What is the advantage of performing LU-factorization of matrix A and solving the system $Ax = b$ at separate stages?

   (b) Suppose we want to solve the system $Ax = b$. If A and b are exactly representable, what conditions would be needed to ensure an accurate computer solution $\hat{x}$?

   (c) How many free parameters does a quadratic spline with n intervals have?

   (d) What is an advantage of using Newton polynomial representation over monomial polynomial representation? What is an advantage of using monomial polynomial representation over Newton polynomial representation?

   (e) What is an advantage of using Newton-Cotes rules over Gauss rules? What is an advantage of using Gauss rules over Newton-Cotes rules?

2. The roots of $ax^2 + bx + c = 0$, when $a \neq 0$, are

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

   (a) [4] Using four-digit rounding arithmetic, apply this formula to the equation $x^2 + 62.10x + 1 = 0$, whose roots are approximately

$$x_1 = -0.01610723 \text{ and } x_2 = -62.08390.$$

   (b) [2] Explain why one of the roots is less accurate than the other.

   (c) [4] Suppose that one root is accurately computed. How would you use this fact to compute the other root? Use this method to improve the accuracy of one of the roots found in (a).

3. (a) [5] Describe an effective method to solve the linear system $ABCx = b$ for the unknown vector x where A, B, C are $n \times n$ nonsingular matrices and b is an $n \times 1$ vector. What is the flops count?

   (b) [5] Let $p_n(x) = a_0 + a_1 x + \ldots + a_n x^n$. Give an algorithm that uses Horner's rule to evaluate $p_n'(x)$.

4. (a) [6] Let $f(x) = e^x - x - 1$. Apply Newton's method to f for 3 iterations with initial guess $x_0 = 1$.

   (b) [4] What is the likely order of convergence β and asymptotic error constant κ?

5. (a) [4] Let $p_3(x)$ be the polynomial interpolant of $f(x) = e^x$ on the points $(-1.0, 0.3679), (-0.5, 0.6065), (0.5, 1.6467), (1.0, 2.7183)$. Find $p_3(x)$ using any method discussed in class.

   (b) [6] Find the best bound that you can on the approximation error

$$\max_{-1 \le x \le 1} |f(x) - p_3(x)|.$$

   (c) [2] Suppose the interpolating points were taken to be the roots of the Chebychev polynomial $T_4(x)$. Derive a new, tighter, upper bound on the approximation error.

   (d) [3] Suppose now that a Hermite interpolant of $f(x)$ was determined using the original nodes $\{-1.0, -0.5, 0.5, 1.0\}$. Using your results from (b), find the best bound you can on the Hermite approximation error

$$\max_{-1 \le x \le 1} |f(x) - p_7(x)|.$$

   (e) [5] Find the **clamped** cubic spline $S(x)$ that interpolates $(-1.0, 0.3679)$, $(-0.5, 0.6065), (0.5, 1.6467), (1.0, 2.7183)$ with $S'(-1.0) = p_3'(-1.0)$ and $S'(1.0) = p_3'(1.0)$.

6. We would like to use adaptive quadrature based on composite Simpson's rule to find an approximation to the integral

$$I = \int_0^1 \sqrt{x}\,dx$$

correct to an absolute error of 5×10^{-4}.

   (a) [4] Let S_n be an approximation of I by composite Simpson rule with n panels and $E_n = I - S_n$ be its respective error. Derive an estimation of E_{2n} based on S_n and S_{2n}.

(b) [3] Where do you expect that it will be easy to approximate I? Where will it be difficult?

(c) [8] We start by subdividing the interval of integration into 2 panels. Use 2-panel composite Simpson to approximate $I_1 = \int_0^{0.5} \sqrt{x}dx$ and $I_2 = \int_{0.5}^1 \sqrt{x}dx$. Without computing the actual integrals, find the approximation error for each. Which, if any, approximation requires refinement?

7. [5] Describe in detail how you would approximate the improper integral

$$I = \int_0^\infty \frac{1}{1+x^4}dx$$

to within 10^{-6}. You do not need to compute the integral.

Total Pages=3

Total Marks=80

Have a good summer!