CSC336 Assignment 4 Due Mon Dec 2

Please **underline** your last name. The assignment is due at the beginning of tutorial.
For all the programming questions, hand in the script file and plot when applicable.

1. Consider the quadrature rule of the form

$$\int_0^1 f(x)dx \approx Af(0) + Bf(\frac{1}{4}) + Cf(\frac{1}{2}) + Df(\frac{3}{4}) + Ef(1)$$

   (a) [10] Find A, B, C, D and E so that the rule above has maximum polynomial degree. What is the polynomial degree?

   (b) [5] Show how the rule in (a) can be transformed into a rule for approximating $\int_a^b f(x)dx$.

   (c) [5] Use the rule in (a) to approximate $\ln(2)$. Compare your answer to the correct value and compute the error.

2. Consider the integral $I = \int_0^4 \frac{1}{2x+8\sqrt{x}}dx$.

   (a) [5] Find the exact value of I by hand.

   (b) [10] Write a short MATLAB `function Q = Trap(fname, s, a, b)` that implements the Compound Trapezoidal rule. Here, $fname$ is the function name, s is the number of panels used, a, b are the end points of the interval of integration Q is the approximation. Determine theoretically the values of s and h required to approximate I to within 10^{-5} and compute the approximation.

3. (a) [8] Write a MATLAB `function [Q, E] = SimpleSimp(fname, a, b)` that uses the Simpson's rule with 2 panels to return an approximation of the integral of a function in the closed interval $[a, b]$ and an estimated error of the approximation. Here, $fname$ is the function name, Q is the approximation and E is the estimated error.

   (b) [14] Now, write a MATLAB `function Q = AdaptSimp(fname, a, b, tol, lv)` that implements the adaptive quadrature based on the Simpson's rule as outlined in lecture 10.4. Here, tol is the target error of the integral approximation, lv is the level of recursion, and Q is the integral approximation. In addition, for each recursion level, print Q, and E from the call to `SimpleSimp`, tol, lv and a, b. The recursion level should not exceed 10.

   Use your adaptive quadrature rule to estimate the value of

$$I = \int_0^1 (1-x)^{1/4}dx.$$

   Your final result should be accurate to within an estimated absolute error of 0.001. Where do you expect that it will be easy to approximate I? Where would it be difficult? Does the output agree with your predictions?

(c) [3] Find the exact value of I and determine whether or not your result from (b) is accurate to within an absolute error of 0.001.

4. [20] Write a MATLAB function `Q = dbquad(fname, a, b, n)` that uses the Compound Simpson's rule with n panels in both directions to return an estimate of

$$I = \int_a^b \left(\int_x^{2x} f(x,y)dy \right) dx$$

where $fname$ is the function name. Test it for $a = 0, b = 1, n = 6$ and the function $f(x,y) = y^2 + x^3$. Compare your result to the exact answer.

5. [Bonus 5] Calculate the polynomial of degree ≤ 3 that minimizes

$$\int_{-1}^{1} [\cos(\pi x) - p(x)]^2 dx$$

over all polynomials $p(x)$ of degree ≤ 3. Use a basis of Legendre polynomials. Plot your $p(x)$ and $\cos(\pi x)$ over $[-1, 1]$ in a single figure to get a picture of the accuracy of your approximation.

Total Marks 80