CSC336 Assignment 3 Due Mon Nov 11

Please **underline** your last name. The assignment is due at the beginning of tutorial.
For all the programming questions, hand in the script file and plots when applicable.

1. (a) [10] Let $p_3(x)$ be the polynomial interpolant of $f(x) = x^5$ on the points $\{(-2, f(-2)), (-1, f(-1)), (1, f(1)), (2, f(2))\}$. Find the Lagrange and Newton forms of the interpolating polynomial for the above data. Write both of the polynomials in monomial form to verify that they are the same function.

   (b) [7] Find the best bound that you can on the approximation error
   $$\max_{-2 \le x \le 2} |f(x) - p_3(x)|.$$

   (c) [3] Suppose that the interpolating points were taken to be the roots of the Chebychev polynomial $T_4(x)$ for the interval $[-2, 2]$. Derive a new, tighter, upper bound on the approximation error.

2. [10] Determine the scalars $\alpha_i, \beta_i, \gamma_i, i = 0, 1, 2$ so that the following is a quadratic B-spline, defined with respect to the knots 0,1,2,3.
   $$B(x) = \begin{cases} \alpha_0 + \alpha_1 x + \alpha_2 x^2 & 0 \le x \le 1 \\ \beta_0 + \beta_1 (x-1) + \beta_2 (x-1)^2 & 1 \le x \le 2 \\ \gamma_0 + \gamma_1 (3-x) + \gamma_2 (3-x)^2 & 2 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$
   (Write down all the relations that $B(x)$ has to satisfy in order to be a quadratic B-spline and solve for $\alpha_i, \beta_i, \gamma_i, i = 0, 1, 2$.)

3. [25] MATLAB provides functions `polyfit` and `polyval`, that can be used to construct and evaluate polynomial interpolants. More specifically, if `x` and `y` are vectors of length $n+1$, then `yv = polyval(polyfit(x, y, n), xv)` constructs the polynomial of degree n that passes from the data points specified by `x` and `y`, and evaluates it at the points with abscissae `xv`, returning the values in `yv`.

   Consider the functions $R(x) = \frac{1}{1+25x^2}$ and $A(x) = e^{-25x^2}$ in the interval $[-1, 1]$. For each of the two functions and for $n = 4, 8, 16, 32$ do the following:
   • Call the `polyfit` and `polyval` functions to construct an interpolant for n data points, chosen once to be equidistant, and once to be Chebychev points in $[-1, 1]$.
   • Plot the two interpolants and the exact function in one plot. For your plot, use the command `xv = linspace(-1,1,101)` to generate 101 equally spaced points in $[-1, 1]$.
   • Compute the maximum error in absolute value for each interpolant over all the equally spaced points in `xv`.

Use MATLAB to table the error results:

Function	data chosen	n=4	n=8	n=16	n=32

Here, Function is either $R(x)$ or $A(x)$ and data chosen is either *Chebychev* or *Equal*. Comment on your results and give some explanations.

4. (a) [25] We would like to construct a natural cubic spline. First write the MATLAB `function [a, b, c, d] = NSpline(x,y)`. Here, x, y are column vectors corresponding to the set of interpolating points (x_i, y_i), and the column vectors a, b, c, d are the set of coefficients for a natural spline as outlined in Prof Fairgrieve's note.

   (b) [10] Now write the MATLAB `function yv = NSplineEval(a,b,c,d,x,xv)`. Here, a, b, c, d, x are described as in (a), xv is a column vector of evaluation points, and yv is a column vector whose entries correspond to the natural spline's evluation of the points on xv. Test out your natural spline on the function $f(x) = \sin(x)$. Let $x = [0, \pi/3, 3\pi/4, 3\pi/2, 11\pi/6, 2\pi]^T$. Output the values a, b, c, d. Plot both $f(x)$ and the natural spline on 101 equally spaced points in the interval $[0, 2\pi]$ on the same graph.

Total Marks 90