CSC336 Assignment 1 Due Mon Sept 30

Please **underline** your last name. The assignment is due at the beginning of tutorial.
For each programming question, hand in the script file and output.

1. [10] For each of the following functions find the condition number and determine the values of x where the condition number is large.
$$(x-1)^a, \quad \ln x, \quad \sin x, \quad e^x, \quad e^x/x.$$

2. [10] Assume that MATLAB produces an operation error of δ each time it performs an operation. Derive a tight error bound μ for the MATLAB evaluation of $(x+y)z$. Assume that x, y and z each have a relative round-off error of 2^{-12}. What should δ be if you want $\mu \leq 2^{-10}$?

3. [10] Consider using the Taylor series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ to approximate e^x. What are some possible problems of using such a series? Write an efficient pseudo-code that uses the above Taylor series to approximate e^x.

4. [10] Let F be a set of normalized positive floating point numbers in a base-2 computer with 10-bit mantissas and exponent range $[-20, 20]$. What is the binary form of the number $x = 2/3$? Find the largest floating point number x_l in F such that $x_l \leq x$. Find the smallest floating point number x_r in F such that $x \leq x_r$. Which of these should be taken to be $fl(x)$? What are the absolute round-off error and the relative round-off error in representing x by $fl(x)$?

5. Consider the sequence of real numbers defined inductively by
$$\begin{cases} x_0 = 1, \quad x_1 = \frac{1}{3} \\ x_{n+1} = \frac{13}{3}x_n - \frac{4}{3}x_{n-1}, \quad n \geq 1 \end{cases} \tag{1}$$
   (a) [5] Prove that $x_n = (1/3)^n$.
   (b) [10] Write an efficient MATLAB script to evaluate (1). Print the values of x_n and $(1/3)^n$ for $n = 2 \ldots 15$. You may use the command `format long` to display the digits in long format. Comment on the result.

6. [15] Consider the three functions
$$\begin{aligned} f(x) &= x^8 - 8x^7 + 28x^6 - 56x^5 + 70x^4 - 56x^3 + 28x^2 - 8x + 1 \\ g(x) &= (((((((x-8)x+28)x-56)x+70)x-56)x+28)x-8)x+1 \\ h(x) &= (x-1)^8. \end{aligned}$$
   Calculate each function in MATLAB in a straightforward way without rearranging or factoring. Use the command `x = linspace(0.99,1.01,101)` to generate 101 equally spaced points in $[0.99, 1.01]$. Use the `plot` command to plot all three functions evaluated on x on the same grid. What is the relationship of the three functions? Comment on the result.

Total Marks 70