Quadrature rules (simple)

Quadrature rule	n	d	respective poly.interp.	approx. $Q(f)$ to integral $I(f)$	error $I(f)-Q(f)$
rectangle	1	0	constant	$(b-a)f(a)$	$f'(\eta)\frac{(b-a)^2}{2}$
midpoint	1	1	constant	$(b-a)f(m)$	$f''(\eta)\frac{(b-a)^3}{24}$
trapezoid	2	1	linear	$\frac{b-a}{2}(f(a)+f(b))$	$-f''(\eta)\frac{(b-a)^3}{12}$
Simpson	3	3	quadratic	$\frac{b-a}{6}(f(a)+4f(m)+f(b))$	$-f^{(4)}(\eta)\frac{(b-a)^5}{2880}$
corr. trap.	4	3	Herm. cub.	$\frac{b-a}{2}(f(a)+f(b))$ $+\frac{(b-a)^2}{12}(f'(a)-f'(b))$	$f^{(4)}(\eta)\frac{(b-a)^5}{720}$
gen. NC	n	$\geq n-1$	deg $\leq n-1$	$\sum_{i=1}^{n} w_i f(x_i)$	$f^{(d+1)}(\eta)\frac{(b-a)^{d+2}}{\mathrm{K}}$
2-pt Gauss	2	3	linear	$\frac{b-a}{2}[f(\frac{a+b}{2}-\frac{b-a}{2\sqrt{3}})$ $+f(\frac{a+b}{2}+\frac{b-a}{2\sqrt{3}})]$	$f^{(4)}(\eta)\frac{(b-a)^5}{4320}$
gen. Gauss	n	$2n-1$			$f^{(d+1)}(\eta)\frac{(b-a)^{d+2}}{C}$

n number of data (and number of function evaluations), including derivative data,
d polynomial degree (degree of accuracy) of the quadrature rule,
η an unknown point in $[a,b]$, in general different for each rule,
$m=\frac{a+b}{2}$ midpoint of the interval and
K, C some constants, in general different for each rule.

Compound quadrature rules

Quadrature rule	n	d	respect. pc. poly.interp.	approx. $Q(f)$ to integral $I(f)$	error $I(f)-Q(f)$
rectangle	s	0	constant	$h\sum_{i=0}^{s-1} f(a+ih)$	$f'(\eta)\frac{h}{2}(b-a)$
midpoint	s	1	constant	$h\sum_{i=1}^{s} f(a+(i-\frac{1}{2})h)$	$f''(\eta)\frac{h^2}{24}(b-a)$
trapezoid	$s+1$	1	linear	$\frac{h}{2}(f(a)+2\sum_{i=1}^{s-1} f(a+ih)+f(b))$	$-f''(\eta)\frac{h^2}{12}(b-a)$
Simpson	$2s+1$	3	quadratic	$\frac{h}{6}[f(a)+2\sum_{i=1}^{s-1} f(a+ih)$ $+4\sum_{i=1}^{s} f(a+(i-\frac{1}{2})h)+f(b)]$	$-f^{(4)}(\eta)\frac{h^4}{2880}(b-a)$
corr. trap.	$s+3$	3	Herm. cub.	$\frac{h}{2}(f(a)+2\sum_{i=1}^{s-1} f(a+ih)+f(b))$ $+\frac{h^2}{12}(f'(a)-f'(b))$	$f^{(4)}(\eta)\frac{h^4}{720}(b-a)$
gen. NC		d			$f^{(d+1)}(\eta)\frac{h^{d+1}}{\mathrm{K}}(b-a)$

n number of data (and number of function evaluations), including derivative data,
d polynomial degree of the quadrature rule,
η an unknown point in $[a,b]$, in general different for each rule,
$h=\frac{b-a}{s}$ the step-size of each panel,
s the number of panels and
K some constant, in general different for each rule.

Comparison between Newton-Cotes and Gauss rules

Property	Newton-Cotes	Gauss
points equidistant	yes	no
points rational	yes	mostly no
weights positive	not all (for $n \geq 10$ some weights are always negative)	yes
weights rational	yes	mostly no
end-points included	yes or no (closed or open)	no (open)
polynomial degree (degree of accuracy) with $n+1$ points	n or $n+1$	$2n+1$
interpolatory	yes	yes
re-use of function evals when doubling the number of panels	often yes	mostly no