SCI 199 (Beautiful Algorithms)

Assignment #3 Due: Dec. 6, 2001 (in tutorial)

Note: In all questions, the better the algorithm, the better the mark.

1. Assume you have a function FLIP which returns a random integer that is either 0 or 1 (i.e. we're simulating the flip of a coin). Design an algorithm which uses FLIP to pick a card at random from a normal 52 card deck.

2. Consider the matrix chaining algorithm presented in class.

   (a) Show how the algorithm works (i.e. fill out the dynamic programming table) for the following sequence of matrices:
   (30,35) * (35,15) * (15,5) * (5,10) * (10,20) * (20,25)

   (b) The algorithm presented in class calculates the minimum number of operations needed to calculate the product of the matrices but does not indicate the order in which the matrix multiplications should take place. Modify this algorithm so that it produces the order of the multiplications.

   (c) Show how your modified algorithm works on the above example.

3. The "Sabena" problem discussed in class requires a permutation of the integers from 0 to 9 so that for each i from 1 to 10, the number formed by the first i digits is divisible by i.

   (a) Continue the backtracking started in class to find all permutations (if any) that satisfy the above condition. The less backtracking the better.

   (b) **EXTRA:** For each j from 1 to 9, solve the Sabena problem for the integers from 1 to j. (Note that the (a) part is concerned with the $j = 10$ case.) For example, the $j = 5$ case requires a permutation of the integers from 0 to 4 so that for each i from 1 to 5, the number formed by the first i digits is divisible by i. It is possible that for some values of j a solution is not possible.

4. Now consider the "Air Canada" problem (in their most recent magazine): You're given a square of circles where each circle contains an integer. You are required to add edges between the circles such that

   - only horizontal, vertical or diagonal edges are added
   - the number of edges touching a circle equals the integer inside the circle
   - no pair of edges cross

   For example, consider the following 4*4 square, and its solution. Note that no backtracking is required to come up with this solution.

   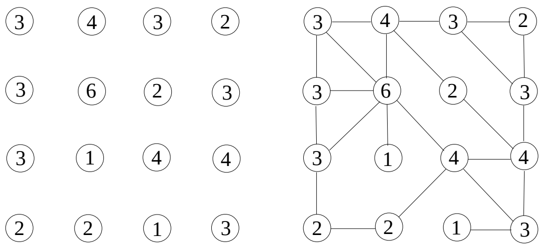

   Figure 1: A 4x4 square with its solution.

   Now consider the following 5*5 square.

   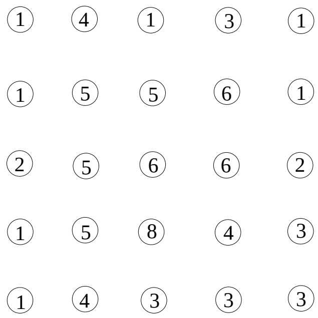

   Figure 2: A 5x5 square.

   (a) Draw in all edges that are forced (i.e. no backtracking is needed).
   (b) Now solve the problem. Briefly describe the backtracking that you've done.
   (c) Is your solution unique? Briefly justify.