SCI 199 (Beautiful Algorithms)

Assignment #7 Due: March 21, 2002 (in tutorial)

Note: In all questions, the better the algorithm, the better the mark.

1. **[5 marks]** Let $G(V, E)$ be a connected graph in which a cost is associated with each edge. Let S be a given strict subset of V (i.e. S is a set of vertices). We want to find a spanning tree T of G such that:
   (i) Every vertex in S is a leaf in T.
   (ii) T has minimum cost among all spanning trees satisfying (i).

   (a) Given $S \subset V$, is there always a spanning tree such that the vertices in S are leaves in T? If so, give a short proof; if not, give a counter-example.

   (b) Assuming such a tree does exist, briefly describe an algorithm to find a minimum weight tree satisfying (i).

2. **[5 marks]** Consider the following algorithm (Floyd's Algorithm) to calculate the Distance Matrix of a given graph (or digraph). Note that the algorithm calculates a series of matrices, $D^0, D^1, \cdots, D^n$, where D^k is a $n * n$ array, $(1 \leq k \leq n)$. We assume that D^0 stores the initial weights on the edges where $w(i, j) = \infty$ if there is no edge from i to j; otherwise, $w(i, j)$ equals the weight of the edge.

```
for k := 1 to n
    for i := 1 to n
        for j := 1 to n
            D^k(i,j) := MIN(D^{k-1}(i,j), D^{k-1}(i,k) + D^{k-1}(k,j))
```

   (a) Trace this algorithm by showing the final values of each of the D^k for the following graph (the same graph that you saw in class to illustrate the Dantzig Algorithm).

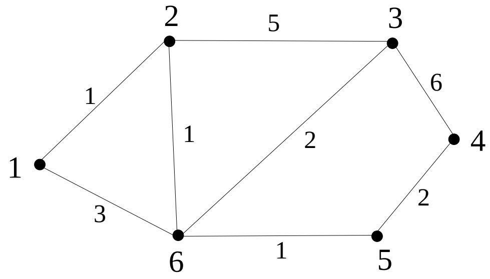

Figure 1:

(b) Briefly describe how the algorithm works. What is the time complexity of the Floyd Algorithm? Briefly justify.

(c) This algorithm will work if there are negative weights on the edges as long as there is not a negative cycle. In fact, this algorithm can be used to determine the presence of a negative cycle in the given graph. How? Briefly justify your answer.
HINT: There is a simple test that can be performed on each $D^k, k > 0$.

3. **[EXTRA]** A couple invites 4 other couples to their home. During the evening, everyone shakes hands with all people they have never met before. Clearly nobody shakes hands with his/her spouse. The hostess goes around and talks to all the other people (i.e. the 9 other people) and determines that each of the other people has shaken hands with a different number of people.

(a) How many hands did the host shake?
HINT: First solve the problem for 1 other couple being invited, then for 2 other couples etc.

(b) If n other couples had been invited, how many hands would the host have shaken?