SCI 199 (Beautiful Algorithms)

Assignment #6 Due: Feb 14, 2002 (in tutorial)

Note: In all questions, the better the algorithm, the better the mark.

1. **[2 marks]** Find two nonisomorphic trees with the same degree sequence. Give a convincing argument that the trees are not isomorphic.

2. **[4 marks]** There are 5 nonisomorphic graphs with the degree sequence 2-2-3-3-4-4. Find any 4 of these graphs and for each pair give a short convincing argument why the pair is not isomorphic.

3. **[4 marks]**

   (a) Suppose that the edge costs are all distinct in an input G to the MST problem. Prove that G has a unique minimum cost spanning tree by giving a brief explanation that follows the Kruskal alg.

   (b) Now suppose that all edge costs are distinct, except $c_1 = c_2$, where $c_1 = c(e_1)$ and $c_2 = c(e_2)$ are the two minimum edge costs. Prove (using the same level of argument as in part (a)) or give a counter-example: G has a unique minimum cost spanning tree.

   (c) Finally suppose that all edge costs are distinct, except $c_1 = c_2 = c_3$, where $c_1 = c(e_1), c_2 = c(e_2), c_3 = c(e_3)$ are the three minimum edge costs. Prove (using the same level of argument as in part (a)) or give a counter-example: G has a unique minimum cost spanning tree.

4. **[EXTRA]**

   (a) Use graph theory to answer the following question. Prove that every set of six people contains (at least) three mutual acquaintances or three mutual strangers.

   In such questions, one usually lets the vertices represent the people and have an edge between a pair of vertices if and only if the corresponding people are acquaintances. Thus a non-edge between a pair of vertices indicates that the people are strangers.

   (b) Is the above question true for every set of five people? Either prove it's true or find a counter-example.