CSC 411: Introduction to Machine Learning

CSC 411 Lecture 22: Reinforcement Learning II

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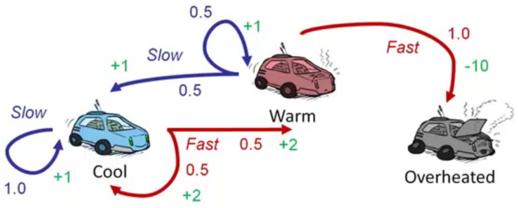
MDP

- Markov Decision Problem (MDP): tuple (S, A, P, γ) where P is

$$P(s_{t+1} = s', r_{t+1} = r' | s_t = s, a_t = a)$$

- Main assumption: Markovian dynamics and reward.
- Standard MDP problems:
  1. **Planning**: given complete Markov decision problem as input, compute policy with optimal expected return



[Pic: P. Abbeel]

Basic Problems

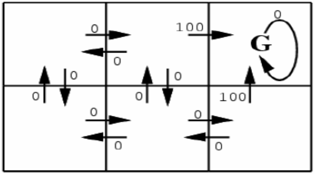
- Markov Decision Problem (MDP): tuple (S, A, P, γ) where P is

$$P(s_{t+1} = s', r_{t+1} = r' | s_t = s, a_t = a)$$

- Standard MDP problems:
  1. **Planning**: given complete Markov decision problem as input, compute policy with optimal expected return
  2. **Learning**: We don't know which states are good or what the actions do. We must try out the actions and states to learn what to do

[P. Abbeel]

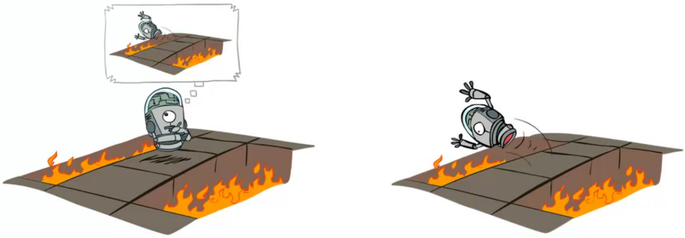
Example of Standard MDP Problem



$r(s, a)$ (immediate reward)

1. **Planning**: given complete Markov decision problem as input, compute policy with optimal expected return
2. **Learning**: Only have access to experience in the MDP, learn a near-optimal strategy

Example of Standard MDP Problem



1. **Planning**: given complete Markov decision problem as input, compute policy with optimal expected return
2. **Learning**: Only have access to experience in the MDP, learn a near-optimal strategy

We will focus on learning, but discuss planning along the way

Exploration vs. Exploitation

- If we knew how the world works (embodied in P), then the policy should be deterministic
  - just select optimal action in each state
- Reinforcement learning is like trial-and-error learning
- The agent should discover a good policy from its experiences of the environment
- Without losing too much reward along the way
- Since we do not have complete knowledge of the world, taking what appears to be the optimal action may prevent us from finding better states/actions
- Interesting trade-off:
  - immediate reward (**exploitation**) vs. gaining knowledge that might enable higher future reward (**exploration**)

Examples

- Restaurant Selection
  - **Exploitation**: Go to your favourite restaurant
  - **Exploration**: Try a new restaurant
- Online Banner Advertisements
  - **Exploitation**: Show the most successful advert
  - **Exploration**: Show a different advert
- Oil Drilling
  - **Exploitation**: Drill at the best known location
  - **Exploration**: Drill at a new location
- Game Playing
  - **Exploitation**: Play the move you believe is best
  - **Exploration**: Play an experimental move

[Slide credit: D. Silver]

Value function

- The value function $V^\pi(s)$ assigns each state the expected reward

$$V^\pi(s) = \mathop{\mathbb{E}}_{a_t, a_{t+i}, s_{t+i}} \left[\sum_{i=1}^{\infty} \gamma^i r_{t+i} | s_t = s \right]$$

- Usually not informative enough to make decisions.
- The Q-value $Q^\pi(s, a)$ is the expected reward of taking action a in state s and then continuing according to π.

$$Q^\pi(s, a) = \mathop{\mathbb{E}}_{a_{t+i}, s_{t+i}} \left[\sum_{i=1}^{\infty} \gamma^i r_{t+i} | s_t = s, a_t = a \right]$$

Bellman equations

- The foundation of many RL algorithms

$$
\begin{aligned}
V^{\pi}(s) &= \mathop{\mathbb{E}}_{a_t, a_{t+i}, s_{t+i}} \left[\sum_{i=1}^{\infty} \gamma^i r_{t+i} | s_t = s \right] \\
&= \mathop{\mathbb{E}}_{a_t} [r_{t+1} | s_t = s] + \gamma \mathop{\mathbb{E}}_{a_t, a_{t+i}, s_{t+i}} \left[\sum_{i=1}^{\infty} \gamma^i r_{t+i+1} | s_t = s \right] \\
&= \mathop{\mathbb{E}}_{a_t} [r_{t+1} | s_t = s] + \gamma \mathop{\mathbb{E}}_{s_{t+1}} [V^{\pi}(s_{t+1}) | s_t = s] \\
&= \sum_{a,r} P^{\pi}(a|s_t) p(r|a, s_t) \cdot r + \gamma \sum_{a,s'} P^{\pi}(a|s_t) p(s'|a, s_t) \cdot V^{\pi}(s')
\end{aligned}
$$

- Similar equation holds for Q

$$
\begin{aligned}
Q^{\pi}(s, a) &= \mathop{\mathbb{E}}_{a_{t+i}, s_{t+i}} \left[\sum_{i=1}^{\infty} \gamma^i r_{t+i} | s_t = s, a_t = a \right] \\
&= \sum_{r} p(r|a, s_t) \cdot r + \gamma \sum_{s'} p(s'|a, s_t) \cdot V^{\pi}(s') \\
&= \sum_{r} p(r|a, s_t) \cdot r + \gamma \sum_{a',s'} p(s'|a, s_t) p(a'|s') \cdot Q^{\pi}(s', a')
\end{aligned}
$$

Solving Bellman equations

- The Bellman equations are a set of linear equations with a unique solution.
- Can solve fast(er) because the linear mapping is a contractive mapping.
- This lets you know the quality of each state/action under your policy - **policy evaluation**.
- You can improve by picking $\pi'(s) = \max_a Q^\pi(s, a)$ - **policy improvement**.
- Can show the iterative policy evaluation and improvement converges to the optimal policy.
- Are we done? Why isn't this enough?
  - Need to know the model! Usually isn't known.
  - Number of states is usually huge (how many unique states does a chess game have?)

Optimal Bellman equations

- First step is understand the Bellman equation for the optimal policy π^*
- Under this policy $V^*(s) = \max_a Q^*(s, a)$

$$
\begin{aligned}
V^*(s) &= \max_a \left[\mathbb{E}\left[r_{t+1} | s_t = s, a_t = a\right] + \gamma \mathop{\mathbb{E}}_{s_{t+1}} \left[V^*(s_{t+1}) | s_t = s, a_t = a\right] \right] \\
&= \max_a \left[\sum_r p(r|a, s_t) \cdot r + \gamma \sum_{s'} p(s'|a, s_t) \cdot V^*(s') \right] \\
Q^*(s, a) &= \mathbb{E}\left[r_{t+1} | s_t = s, a_t = a\right] + \gamma \mathop{\mathbb{E}}_{s_{t+1}} \left[\max_{a'} Q^*(s_{t+1}, a') | s_t = s, a_t = a \right] \\
&= \sum_r p(r|a, s_t) \cdot r + \gamma \sum_{s'} p(s'|a, s_t) \cdot \max_{a'} Q^*(s', a')
\end{aligned}
$$

- Set on nonlinear equations.
- Same issues as before.

Q-learning intuition

- Q-learning is a simple algorithm to find the optimal policy without knowing the model.
- Q^* is the unique solution to the optimal Bellman equation.

$$Q^*(s,a) = \mathbb{E}\left[r_{t+1}|s_t = s, a_t = a\right] + \gamma \mathop{\mathbb{E}}_{s_{t+1}}\left[\max_{a'} Q^*(s_{t+1}, a')|s_t = s, a_t = a\right]$$

- We don't know the model and don't want to update all states simultaneously.
- Solution - given sample $s_t, a_t, r_{t+1}, s_{t+1}$ from the environment update your Q-values so they are closer to satisfying the bellman equation.
  - **off-policy** method: Samples don't have to be from the optimal policy.
- Samples need to be diverse enough to see everything - exploration.

Exploration vs exploitation

- Given Q-value the best thing we can do (given our limited knowledge) is to take $a = \arg\max_{a'} Q(s, a')$ - **exploitation**
- How do we balance exploration with exploitation?
- Simplest solution: ϵ-greedy.
  - With probability $1 - \epsilon$ pick $a = \arg\max_{a'} Q(s, a')$ (i.e. greedy)
  - With probability ϵ pick any other action uniformly.
- Another idea - softmax using Q values
  - With probability $1 - \epsilon$ pick $a = \arg\max_{a'} Q(s, a')$ (i.e. greedy)
  - With probability ϵ pick any other action with probability $\propto \exp(\beta Q(s, a))$.
- Other fancier solutions exist, many leading methods use simple ϵ-greedy sampling.

Q-learning algorithm

Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
    $S \leftarrow S'$;
  until S is terminal

- Can prove convergence to the optimal Q^* under mild conditions.
- Update is equivalent to gradient descent on loss $||R + \gamma \max_a Q(S', a) - Q(s, a)||^2$.
- At optimal Q, the loss is 0.

Bootstrapping

- Another way to think about Q-learning.
- $Q(s, a)$ is the expected reward, can use Monte-Carlo estimation.
- Problem - you update only after the episode ends, can be very long (or infinite).
- Q-learning solution - take only 1 step forward and estimate the future using our Q value - **bootstrapping**.
  - "learn a guess from a guess"
- Q-learning is just one algorithm in a family of algorithms that use this idea.

Function approximation

- Q-learning still scales badly with large state spaces, how many states does a chess game have? Need to save the full table!
- Similar states, e.g. move all chess pieces two steps to the left, at treated as totally different.
- Solution: Instead of Q being a $S \times A$ table it is a parametrized function.
- Looking for function $\hat{Q}(s, a; \mathbf{w}) \approx Q^*(s, a)$
  - Linear functions $Q(s, a; \mathbf{w}) = \mathbf{w}^T \phi(s, a)$.
  - Neural network
- Hopefully can generalize to unseen states.
- Problem: Each change to parameters changes all states/actions - can lead to instability.
- For non-linear Q-learning can diverge.

Deep Q-learning

- We have a function approximator $Q(s, a; \theta)$, standard is neural net but doesn't have to be.
- What is the objective that we are optimizing?
- We want to minimize $\mathbb{E}_{\rho}[||R + \gamma \max_{a'} Q(S', a') - Q(s, a)||^2]$
  - ρ is a distribution over states, depends on θ!
- Two terms depend on Q, don't want to take gradients w.r. to $\gamma \max_a Q(S', a)$
- We want to correct our previous estimation given the new information.

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

Figure: Take from:rll.berkeley.edu/deeprlcourse

- This simple approach doesn't work well as is.

Issues and solutions

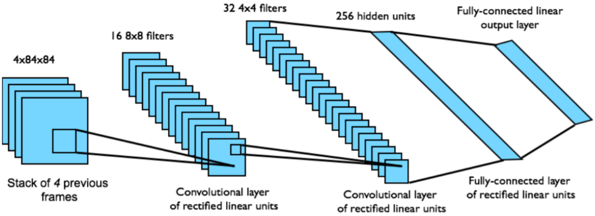
- **Problem**: data in the minibatch is highly correlated
  - Consecutive samples are from the same eposide and probably similar states.
  - Solution: **Replay memory**.
  - You store a large memory buffer of previous (s, a, r, s') (notice this is all you need for Q-learning) and sample from it to get diverse minibatch.
- **Problem**: The data distribution keeps changing
  - Since we aren't optimizing y_i its like solving a different (but related) least squares each iteration.
  - We can stabilize by fixing a **target network** for a few iterations

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly
3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$ using *target* network $Q_{\phi'}$
4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$
5. update ϕ': copy ϕ every N steps

Figure: Take from:rll.berkeley.edu/deeprlcourse

Example: DQN on atari

- Trained a NN from scratch on atari games



- Ablation study

	Replay Fixed-Q	Replay Q-learning	No replay Fixed-Q	No replay Q-learning
Breakout	316.81	240.73	10.16	3.17
Enduro	1006.3	831.25	141.89	29.1
River Raid	7446.62	4102.81	2867.66	1453.02
Seaquest	2894.4	822.55	1003	275.81
Space Invaders	1088.94	826.33	373.22	301.99

RL recap

- Learning from experience not from labeled examples.
- Why is RL hard?
  - Limited feedback.
  - Delayed rewards.
  - Your model effect what you see.
  - Huge state space.
- Usually solved by learning the value function or optimizing the policy (not covered)
- How do you define the rewards? Can be tricky.
  - Bad rewards can lead to **reward hacking**

Q-Learning recap

- Try to find Q that satisfies the optimal Bellman conditions
- **Off-policy** algorithm - Doesn't have to follow a greedy policy to evaluate it.
- **Model free** algorithm - Doesn't have any model for instantaneous reward or dynamics.
- Learns a seperate value for each s, a pair - doesn't scale up to huge state spaces.
- Can scale using a function approximation
    - No more theoretical guarantees.
    - Can diverge.
    - Some simple tricks help a lot.