Abductive Reasoning, Belief Expansion and Nonmonotonic Consequence*

Maurice Pagnucco Abhaya C Nayak Norman Y Foo
email: {morri | abhaya | norman}@cs.su.oz.au
Knowledge Systems Group,
Basser Department of Computer Science,
Madsen Building F09, University of Sydney,
NSW, 2006, Australia.
fax : +61-2-351-3838 tel : +61-2-351-3423

Abstract

When dealing with a situation in which new information is to be incorporated into an epistemic state, most belief revision frameworks, including the AGM, incorporate solely this new information. A more natural proposal is that the agent seeks some explanation or justification first and attempts to incorporate the new information together with its justification into the agent's current epistemic state.

Pagnucco, Nayak and Foo model this proposal by developing a belief change operator known as abductive expansion which adds abductive inference to the belief expansion process. They develop rationality postulates for abductive belief expansion and provide a construction in terms of selection functions. However, this account lacks a proper semantic treatment.

In this paper we present a formal sphere semantics for abductive expansion. By imposing an ordering over the possible worlds that are consistent with current beliefs we demonstrate a method for selecting those worlds consistent with the abductively expanded epistemic state. We also investigate an epistemic entrenchment style ordering over formulae which can be used to determine those formulae to be incorporated into the new epistemic state. This ordering has important consequences for expectation orderings and nonmonotonic inference. Using observations first made by Gärdenfors & Makinson and Williams we indicate how this entrenchment may be used for default reasoning. We also translate the abductive expansion postulates into conditions on a nonmonotonic consequence relation $\mid\!\sim$.

Keywords: Abductive expansion, Abductive reasoning, Belief revision, Nonmonotonic consequence.

1 Introduction

The area of *Belief Revision* is concerned with the study of how an agent modifies its current epistemic (or belief) state upon receipt of some epistemic input. This epistemic input may need to be incorporated into the current epistemic state or removed from it. It is the incorporation of new information with which we shall concern ourselves here.

In the framework developed by Alchourrón, Gärdenfors and Makinson [1][4] (henceforth the AGM framework), upon which we base our current exposition (and in many other frameworks for belief revision), new information that is to be incorporated into an epistemic state is usually added "as is" without any attempt to add further information beyond that which deductively follows from the new information and the current epistemic state. In this paper we pursue a proposal that we consider to be more natural: an agent does not seek to solely incorporate any new information but first attempts to find some reason or justification for the new information and then incorporates this justification, together with the new information, into its current epistemic state. There is evidence to suggest that a similar process takes place in human agents (e.g., [18]). We stress, however, that we are not interested

* To appear in the Proceedings of the ICLP'95 Joint Workshop on Deductive Databases and Logic Programming and Abduction in Deductive Databases and Knowledge-based Systems, Shonan Village Center, Japan, June 1995.

in developing a psychological theory here but, rather, are more interested in the normative aspects of this type of belief change.

Our objective is to use the notion of *abductive inference* to model this proposal. Abductive inference (or simply *abduction*) is a fundamental form of logical inference which attempts to determine plausible "explanations" for data with respect to certain domain knowledge. An abductive inference proceeds by deriving hypotheses which are able to prove the new data when added to the domain theory. Abduction is in fact sometimes referred to as "inference to the best explanation" [8]. For example, suppose we know that when it rains the grass gets wet. If you notice that the grass is wet, then you might hypothesise that it has rained for this would account for your observation. Of course, there are other possibilities; the sprinkler may have been turned on, for instance. Without the benefit of further information you may even wish to simply assume that either it rained or the sprinkler was on.

In the AGM framework, epistemic states are deductively closed. Three types of belief change operators from a current epistemic state K to new epistemic state K'_α are considered: belief expansion (K^+_α) — incorporation of epistemic input α into K without retraction of existing beliefs; belief contraction (K^-_α) — removal of existing belief α from K; and, belief revision (K^*_α) — incorporation of new belief α into K with possible retraction of existing beliefs. In this paper we adopt a view of belief revision based on the AGM framework and attempt to supplement the underlying deductive mechanism with abduction in an effort to model the proposal outlined above. We do this by considering an AGM style belief expansion operator known as *abductive expansion* developed by Pagnucco, Nayak and Foo [15]. This operator can be understood as working in the following fashion: an agent does not expand its current epistemic state solely with respect to the epistemic input but, instead, attempts to incorporate an abduction of the epistemic input (with respect to the current epistemic state), if one is to be found, into its epistemic state and then closes it deductively.

The aim of this paper is to present a slightly simplified version of the rationality postulates for abductive expansion presented by Pagnucco *et al.* [15], review one of their AGM-like constructions for abductive expansion based on selection functions, and, most importantly, develop constructions based on systems of spheres and epistemic entrenchment. We also discuss connections between abductive expansion and default reasoning and translate the rationality postulates into conditions on a (nonmonotonic) consequence relation $\mid\!\sim$ in the manner of Makinson and Gärdenfors [14].

The main reason behind the consideration of belief expansion rather than revision is that the definition of abduction we shall adopt requires an abduction to be consistent with the domain theory. Therefore, if an abduction of an epistemic input is to be derived with respect to the current epistemic state, we will never need to perform revision to incorporate this abduction since expansion will not lead to inconsistency. However, if an abduction cannot be found for an epistemic input α, then α will not be incorporated into the new epistemic state; that is, successful incorporation of the new information is not always guaranteed. Abductive revision based on such an expansion will be discussed briefly in our conclusions.

There are, however, other factors motivating our desire to enhance the AGM belief expansion operator. One, mentioned above, is the interest in examining what occurs in the AGM when the underlying deductive mechanism is supplemented by an abductive one. Another concerns the *principle of minimal change* which is considered important to the AGM ([4] p. 8). Expansion is the only AGM belief change operation where this principle is taken with respect to set inclusion of the current epistemic state in the expanded epistemic state. For contraction and revision the principle of minimal change can be interpreted with respect to the sphere semantics introduced by Grove [7]. In fact, for contraction, minimality with respect to set inclusion — known as maxichoice contraction [4] pp. 76–78 — leads to undesirable results. The same holds for revision functions constructed from maxichoice contraction functions via the Levi identity ([4] p. 69). In this paper we show how the principle of minimal change can be interpreted with respect to the sphere semantics discussed here. We also present an abductive entrenchment ordering over formulae which can be thought of as an extension to the AGM epistemic entrenchment ordering [5].

In the following section we present some technical preliminaries including an overview of AGM expansion along with the definition of abduction that we shall be using. In Section 3 we present a synopsis of abductive expansion including postulates and a construction based on selection functions. We then develop constructions for abductive expansion based on systems of spheres and entrenchment similar in style to those for the AGM. We discuss links to default reasoning in Section 6 and translate our postulates into conditions on a (nonmonotonic) consequence relation $\mid\!\sim$ in Section 7. Related work is considered in Section 8. Conclusions and suggestions for future work are made in Section 9.

2 Preliminaries

We adopt a fixed finite language $\mathcal{L}$ with a complete set of boolean connectives and the symbols $\top$ (truth) and $\bot$ (falsum). Formulae in $\mathcal{L}$ will be denoted by lower case Greek characters $\alpha, \beta, \gamma, \ldots$ while sets of formulae from $\mathcal{L}$ will be denoted by upper case Greek characters $\Delta, \Gamma, \ldots$. We identify the underlying logic with its consequence operator $Cn : 2^{\mathcal{L}} \rightarrow 2^{\mathcal{L}}$ (we often write $\Gamma \vdash \alpha$ to mean $\alpha \in Cn(\Gamma)$). The underlying logic is assumed to be supraclassical (i.e., include classical propositional calculus), compact ($\alpha \in Cn(\Gamma)$ *implies* $\alpha \in Cn(\Gamma')$ *for some finite subset* $\Gamma' \subseteq \Gamma$) and to obey the deduction theorem ($\beta \in Cn(\Gamma \cup \{\alpha\})$ *iff* $(\alpha \rightarrow \beta) \in Cn(\Gamma)$). A *theory* is a set of propositions closed under Cn. A set of formulae Γ is *consistent* if and only if for no proposition α, is it the case that $\alpha \wedge \neg\alpha \in Cn(\Gamma)$.

2.1 AGM Framework for Belief Revision

The AGM framework for belief revision [1, 4] considers three types of belief change — belief expansion, contraction and revision — as outlined in the introduction. Epistemic states are represented by sets of formulae closed under Cn which are known as *belief sets* (i.e., belief sets are theories). These sets shall be denoted by upper case Roman characters $H, K, \ldots$. The set of all possible belief sets shall be denoted $\mathcal{K}$. One special element of $\mathcal{K}$ is the absurd belief set $K_{\bot}$ which contains all formulae in $\mathcal{L}$. Epistemic inputs are represented by a single formula from $\mathcal{L}$. The three types of belief change are investigated in terms of rationality postulates [4] and a number of constructions: selection functions [1, 4], systems of spheres [7] and epistemic entrenchment [5].

Since we are primarily concerned with the process of belief expansion we shall, for completeness, reproduce the relevant rationality postulates (their motivation may be found in [4]):

(K^+1)	K^+_α is a belief set	(closure)
(K^+2)	$\alpha \in K^+_\alpha$	(success)
(K^+3)	$K \subseteq K^+_\alpha$	(inclusion)
(K^+4)	If $\alpha \in K$, then $K^+_\alpha = K$	(vacuity)
(K^+5)	If $K \subseteq H$, then $K^+_\alpha \subseteq H^+_\alpha$	(monotonicity)
(K^+6)	For all belief sets K and all sentences α, K^+_α is the smallest belief set that satisfies (K^+1) — (K^+5)	(minimality)

The following representation theorem characterises expansion functions in terms of Cn.

Theorem 2.1 (AGM [4])
The expansion function $+$ *satisfies* (K^+1) — (K^+6) *iff* $K^+_\alpha = Cn(K \cup \{\alpha\})$.

2.2 Abduction

We adopt a definition of abduction similar to ones commonly found in the literature [9, 16, 17].

Definition 2.1 *An abduction of a formula α with respect to a domain theory Γ is a formula β such that:*

(i) $\Gamma \cup \{\beta\} \vdash \alpha$*;*

(ii) $\Gamma \cup \{\beta\}$ *is consistent (i.e.,* $\Gamma \cup \{\beta\} \not\vdash \bot$*).*

For example, suppose $\Gamma = \{rain \rightarrow wet\_grass,\ sprinkler \rightarrow wet\_grass,\ cloud \wedge dewpoint\_reached \rightarrow rain\ \}$ and $\alpha = wet\_grass$ then, possible abductions of α with respect to Γ include $rain$, $sprinkler$, $rain \wedge sprinkler$, $rain \vee sprinkler$ and $cloud \wedge dewpoint\_reached$.

We maintain that this definition imposes a minimal set of conditions that characterise an abduction. Since we are interested in a general framework capturing all expansions via abduction, we do not impose any further restrictions on abductions here. Different forms of restrictions, however, may be of some interest; for instance, criteria related to syntax, minimality, triviality and specificity (see [16] for an overview).

3 Abductive Expansion

In this section we begin our investigation of the proposed belief change operation of abductive expansion in a manner analogous to the development of expansion, contraction and revision within the AGM framework. We begin with an overview of Pagnucco *et al.* [15] presenting a definition, simplified set of rationality postulates and a construction in terms of selection functions.

The general idea behind abductive expansion is that an agent attempts to find an abduction for new information α with respect to its current belief state K and then takes the deductive closure of this abduction and the current epistemic state to determine the new (abductively expanded) epistemic state. An abductive expansion function, then, is assumed to be a function from pairs of belief sets and formulae to belief sets ($\oplus : \mathcal{K} \times \mathcal{L} \rightarrow \mathcal{K}$). We use the notation $K^{\oplus}_{\alpha}$ to denote the abductive expansion of K by α.

As in the AGM, we are interested in determining the nature of the abductively expanded epistemic state. The notion we have just outlined may be defined as follows:

Definition 3.1 $K^{\oplus}_{\alpha}$ *is an abductive expansion of K by α iff*

$$K^{\oplus}_{\alpha} = \begin{cases} Cn(K \cup \{\beta\}) & \text{for some } \beta \in \mathcal{L} \text{ such that:} \\ & \quad (i)\ K \cup \{\beta\} \vdash \alpha \\ & \quad (ii)\ K \cup \{\beta\} \not\vdash \bot \\ & \\ K & \text{if no such } \beta \text{ exists} \end{cases} \tag{Def AbExp}$$

We shall now present and briefly motivate a number of rationality postulates for abductive expansion. The interested reader is referred to [15] for further details.

3.1 Postulates for Abductive Expansion

($K^{\oplus}1$) For any sentence α and any belief set K,
$K^{\oplus}_{\alpha}$ is a belief set (closure)

An abductive expansion function takes a belief set K and a formula α and returns a new belief set $K^{\oplus}_{\alpha}$. Such a function determines the new epistemic state of an agent given that K is its current epistemic state and it has acquired new information α.

(K$^{\oplus}$2) If $\neg\alpha \notin K$, then $\alpha \in K^{\oplus}_{\alpha}$ (limited success)

If the new information does not conflict with current beliefs (i.e., an abduction for it can be found), then it should be incorporated into the resulting epistemic state.

(K$^{\oplus}$3) $K \subseteq K^{\oplus}_{\alpha}$ (inclusion)

Abductive expansion does not result in the retraction of any currently held beliefs.

(K$^{\oplus}$4) If $\neg\alpha \in K$, then $K^{\oplus}_{\alpha} = K$ (failure)

In the limiting case where the new information conflicts with currently held beliefs and therefore no abduction can be found, no action is taken by the agent in an attempt to maintain consistency[1].

(K$^{\oplus}$5) If $\neg\alpha \notin K$, then $\neg\alpha \notin K^{\oplus}_{\alpha}$ (consistency)

The abductive expansion does not introduce inconsistency.

These first five postulates are referred to as the *basic postulates*. A representation theorem shows their connection to the previous definition.

Theorem 3.1 *The abductive expansion function* $\oplus$ *satisfies* (K$^{\oplus}$1)—(K$^{\oplus}$5) *iff it satisfies* (Def AbExp).

We also consider the following postulate.

(K$^{\oplus}$6) If $K \vdash \alpha \leftrightarrow \beta$, then $K^{\oplus}_{\alpha} = K^{\oplus}_{\beta}$ (strong extensionality)

If, relative to the agent's beliefs, two inputs are considered to carry the same informational content, then their relevant abductive expansions will be identical. It is also possible to consider abductive expansions satisfying the weaker postulate

(WE) If $\vdash \alpha \leftrightarrow \beta$, then $K^{\oplus}_{\alpha} = K^{\oplus}_{\beta}$ ((weak) extensionality)

However, we consider strong extensionality, as it allows us to obtain constructions corresponding more closely to those for the AGM. This is in keeping with our aim to incorporate abduction into the AGM framework.

Another postulate we do not consider is the following vacuity postulate.

(V) If $\alpha \in K$, then $K^{\oplus}_{\alpha} = K$ (vacuity)

The reason such a postulate may seem plausible is that, if we already believe α, then there is no need to explain it in order to incorporate it into our current beliefs (cf. Boutilier and Becher's [3] factual explanations). Moreover, when $\alpha \in K$, then $\beta \rightarrow \alpha \in K$ for every $\beta \in \mathcal{L}$ and therefore any formula consistent with K and α is a possible abduction. Such a restriction is not absolutely necessary for a minimal set of rationality postulates characterising abductive expansion so we do not include it among the postulates here. However, we could add it to our set of postulates with only minor alterations to the constructions presented here[2]. We shall return to it briefly when we translate our postulates into conditions on a nonmonotonic consequence relation $\mid\!\sim$ in Section 7.

[1] An alternative, closer in spirit to the AGM framework, would be to make $K^{\oplus}_{\alpha} = K_{\perp}$ in this case and modify (K$^{\oplus}$2) to always guarantee success. However, we have chosen to place more importance on maintaining consistency rather than guaranteeing success as this seems closer in spirit to the abductive process. See also the comments regarding abductive revision in the conclusion.

[2] It would only represent an additional boundary case.

3.2 Selection Functions

A construction for abductive expansion operators satisfying the postulates above can be given in terms of selection functions applied to the set of maximally consistent supersets of K implying α. This is developed in a manner analogous to an AGM construction for contraction applying selection functions to the set of maximally consistent subsets of K failing to imply α [1, 4].

We note that $K_\alpha^\oplus$ is some superset of K containing α and that any such superset that is closed under Cn can be obtained through the intersection of certain maximally consistent supersets of K implying α. These supersets can be defined as follows.

Definition 3.2 *(Maximally Consistent Superset that Implies α)*
A set K' is a maximally consistent superset of a set K that implies α *iff*

(i) $K \subseteq K'$
(ii) $\alpha \in Cn(K')$
(iii) $Cn(K') \neq K_\perp$
(iv) There is no $K'' \supset K'$ satisfying (i), (ii) and (iii)[3]

The set of all maximally consistent supersets of K implying α is denoted $K\top\alpha$.

Observation 3.2 *Let K be a belief set and $\alpha \in \mathcal{L}$. Any $K' \in K\top\alpha$ is a belief set.*

To determine $K_\alpha^\oplus$ for any K and α we can apply a selection function γ to $K\top\alpha$ which, intuitively, selects the "best" elements from $K\top\alpha$ and take the intersection of these "best" elements. A selection function γ is in fact with respect to some belief set K[4]. Therefore, with this construction in mind, an abductive expansion function $\oplus$ may be modelled by a family of selection functions; one for each $K \in \mathcal{K}$.

An expansion function based on such a selection function γ selecting some subset of $K\top\alpha$ is referred to as a *partial meet abductive expansion function* and is defined by the rule

Definition 3.3 *(Def Part)*

$$K_\alpha^\oplus = \begin{cases} \bigcap\gamma(K\top\alpha) & \textit{whenever } K\top\alpha \textit{ is nonempty} \\ K & \textit{otherwise} \end{cases}$$

A partial meet abductive expansion function is exactly characterised by the postulates above as evidenced by the following representation theorem.

Theorem 3.3 *Let $\oplus$ be an abductive expansion function. For every belief set K, $\oplus$ is a partial meet abductive expansion function* iff *$\oplus$ satisfies postulates* ($K^\oplus 1$)—($K^\oplus 6$) *for abductive expansion over K.*

We shall briefly mention two special types of selection function. If the selection function γ always returns a single element from $K\top\alpha$, the resulting expansion function is referred to as a *maxichoice abductive expansion function*. Alternatively, if γ returns all elements of $K\top\alpha$, the resulting expansion function is referred to as a *full meet abductive expansion function*.

Lemma 3.4 *Any maxichoice or full meet abductive expansion function satisfies* ($K^\oplus 1$)—($K^\oplus 6$).

In fact, in the case where the new information is consistent with the current epistemic state, full meet abductive expansion corresponds to AGM expansion. Hence, we can consider AGM expansion to be a special type of abductive expansion (viz., where, in a sense, the "trivial" abduction α is always selected).

[3]That is, K' can be identified with a $K \cup \{\alpha\}$ *"world"*. See also Section 4.
[4]It may be more informative to write γ_K though we omit the subscript with the understanding that γ is relative to some belief set K.

Theorem 3.5 *Let $\oplus$ be an abductive expansion function. For any formula $\alpha \in L$ and belief sets K and H such that $\neg\alpha \notin K$ and $\neg\alpha \notin H$, the operation $\oplus$ is a full meet abductive expansion of K by α* iff *$\oplus$ satisfies postulates* (K$^+$1)—(K$^+$6) *for AGM expansion over K*[5].

Analogously to the AGM, a selection function γ can be defined by imposing an ordering $\preceq$ over the elements of $K \top \alpha$

(Def γ) $\quad \gamma(K \top \alpha) = \{K' \in K \top \alpha : K'' \preceq K' \ for \ all \ K'' \in K \top \alpha\}$

A selection function so defined is called *relational over K*. Furthermore, if $\preceq$ is transitive, the resulting expansion function is referred to as a *transitively relational partial meet abductive expansion function*. The behaviour of this type of abductive expansion function is captured by the following two postulates: which are a simplified (but equivalent) version of those presented by Pagnucco *et al.* [15].

(K$^\oplus$7) $\quad K^\oplus_\alpha \subseteq Cn(K^\oplus_{\alpha\vee\beta} \cup \{\alpha\})$

($K^\oplus$8) $\quad$ If $\neg\alpha \notin K^\oplus_{\alpha\vee\beta}$, then $K^\oplus_{\alpha\vee\beta} \subseteq K^\oplus_\alpha$

They may be motivated by their following consequence.

$$\text{Either } K^\oplus_{\alpha\vee\beta} = K^\oplus_\alpha \text{ or } K^\oplus_{\alpha\vee\beta} = K^\oplus_\beta \text{ or } K^\oplus_{\alpha\vee\beta} = K^\oplus_\alpha \cap K^\oplus_\beta$$

Theorem 3.6 *Let K be any belief set and $\oplus$ be an abductive expansion function defined over K. Then $\oplus$ is a transitively relational partial meet abductive expansion function* if and only if *$\oplus$ satisfies postulates* (K$^\oplus$1)—(K$^\oplus$8) *for abductive expansion over K.*

We now develop two alternative constructions for transitively relational partial meet abductive expansion functions.

4 Systems of Spheres

Grove [7] developed a "sphere semantics" for the AGM framework. He views maximally consistent sets of formulae as "possible worlds" and places an ordering over the set $\mathcal{M}_\mathcal{L}$ of all possible worlds[6]. The possible worlds consistent with K are denoted $[K]$ (i.e., $[K] = \{m \in \mathcal{M}_\mathcal{L} : K \subseteq m\}$ and, if $K = K_\perp$, then $[K] = \emptyset$). Similarly, the possible worlds consistent with a formula α are denoted $[\alpha]$ ($[\alpha] = \{m \in \mathcal{M}_\mathcal{L} : \alpha \in m\}$). We also define a function $th : 2^{\mathcal{M}_\mathcal{L}} \rightarrow K$ which maps possible worlds to belief sets

$$th(X) = \begin{cases} \bigcap\{m \in X\} & \text{for } X \subseteq \mathcal{M}_\mathcal{L} \text{ and } X \neq \emptyset \\ K_\perp & \text{if } X = \emptyset \end{cases}$$

Grove defines a *sphere* to be a set of possible worlds and a *system of spheres centred on K* to be an ordering over worlds in which $[K]$ comprises the innermost sphere and $\mathcal{M}_\mathcal{L}$ (i.e., all possible worlds) the outermost sphere. That is, he essentially orders those worlds inconsistent with K. Grove goes on to show how those worlds consistent with the new epistemic state can be determined in the case of AGM expansion, contraction and revision[7]. In the case of AGM expansion, the worlds consistent with K^+_α (i.e., $[K^+_\alpha]$) are those given by $[K] \cap [\alpha]$. These are exactly the worlds consistent with both K and α. We illustrate this situation in Figure 1(a).

In abductive expansion, since we are adding more information than just α itself, we need to select some subset of the worlds in $[K] \cap [\alpha]$. To achieve this, we introduce a new system of spheres within $[K]$ itself. We can consider $[K]$ to be an "internal" system of spheres and define such a system as follows.

---

[5] Mention of a belief set H here is meant to refer to that present in postulate (K$^+$5) for AGM expansion over K.

[6] Note the similarity with transitively relational selection functions which impose an ordering over all maximally consistent supersets of K implying α.

[7] Actually, he concentrates on revision but the other cases are not difficult to determine. See also [4] for details.

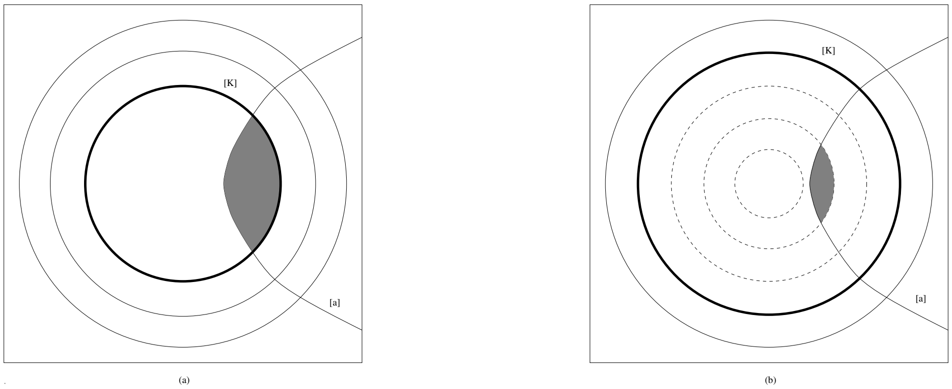

Figure 1: Systems of spheres with (a) $[K_\alpha^+]$ shaded (b) $[K_\alpha^\oplus]$ shaded.

Definition 4.1 *Let $\mathcal{IS}$ be any collection of subsets of $\mathcal{M}_\mathcal{L}$ ($\mathcal{IS} \subseteq 2^{\mathcal{M}_\mathcal{L}}$) and X some subset of $\mathcal{M}_\mathcal{L}$ ($X \subseteq \mathcal{M}_\mathcal{L}$). We say $\mathcal{IS}$ is a system of spheres centred within (or by) X if it satisfies the following conditions:*

(IS1) *$\mathcal{IS}$ is totally ordered by $\subseteq$*
(i.e., for any $U, V \in \mathcal{IS}$, $U \subseteq V$ or $V \subseteq U$)

(IS2) *X is in $\mathcal{IS}$ and, moreover, it is the $\subseteq$-maximal element of $\mathcal{IS}$*
(i.e., for any $U \in \mathcal{IS}$, $U \subseteq X$)

(IS3) *If $\alpha \in \mathcal{L}$ and there is a sphere in $\mathcal{IS}$ intersecting $[\alpha]$, then there is a $\subseteq$-minimal sphere in $\mathcal{IS}$ intersecting $[\alpha]$*
(i.e., if $[\alpha] \cap X \neq \emptyset$, then there is a $U \in \mathcal{IS}$ such that $U \cap [\alpha] \neq \emptyset$ and for any $V \in \mathcal{IS}$, if $V \cap [\alpha] \neq \emptyset$, then $U \subseteq V$)

Note that it follows by (IS3) that $\mathcal{IS}$ has a $\subseteq$-minimal element (i.e., there is some $U \in \mathcal{IS}$ such that $U \subseteq V$ for all $V \in \mathcal{IS}$ — to see this, let α be any tautology).

Condition (IS1) tells us that the internal spheres are nested within one another. (IS2) says that the $\subseteq$-maximal internal sphere contains all worlds in X. (IS3) ensures that, when X and $[\alpha]$ have worlds in common (i.e., they are consistent), for any $\alpha \in \mathcal{L}$, then there is some $\subseteq$-minimal (innermost) sphere in $\mathcal{IS}$ intersecting $[\alpha]$. Such a sphere is denoted $c_{\mathcal{IS}}(\alpha)$.

Having defined an internal system of spheres, $\mathcal{IS}$, we associate a function $f_{\mathcal{IS}} : \mathcal{L} \to 2^{\mathcal{M}_\mathcal{L}}$ with it, defined in the following manner

$$f_{\mathcal{IS}}(\alpha) = \begin{cases} [\alpha] \cap c_{\mathcal{IS}}(\alpha) & \text{if } [\alpha] \cap X \neq \emptyset \\ X & \text{otherwise} \end{cases} \tag{Def $f_{\mathcal{IS}}$}$$

Intuitively, when X and α are consistent, $f_{\mathcal{IS}}(\alpha)$ returns those most preferred worlds in X where α holds. If α does not hold in any X world, then $f_{\mathcal{IS}}(\alpha)$ returns all X worlds.

This internal system of spheres can, together with $f_{\mathcal{IS}}$, now furnish a sphere semantics for abductive expansion which is similar in spirit to that for AGM revision. Referring to Figure 1(b), the structure inside the bold boundary marked $[K]$ represents an internal system of spheres $\mathcal{IS}$ centred within $[K]$. The innermost internal sphere (dashed ring) represents the $\subseteq$-minimal element of $\mathcal{IS}$. It contains the most preferred worlds (i.e., the agent has some predisposition towards these worlds over the other worlds consistent with K). The next internal sphere contains the worlds within the next outermost dashed ring. This continues until the outermost internal sphere, represented by the bold boundary, containing all worlds in $[K]$. The worlds that belong exclusively to this internal sphere (and to no other internal sphere) are the least preferred worlds consistent with K. This means that we adopt the subset minimal change (AGM expansion) only when there is no preference over worlds consistent with K and α.

The possible worlds consistent with the abductively expanded epistemic state $K^{\oplus}_{\alpha}$ are those determined by $f_{\mathcal{IS}}(\alpha)$ (i.e., $[K^{\oplus}_{\alpha}] = f_{\mathcal{IS}}(\alpha)$). That is, those in the intersection of $[\alpha]$ and the innermost sphere intersecting $[\alpha]$ (i.e., $[\alpha] \cap c_{\mathcal{IS}}(\alpha)$) if K and α are consistent. If K and α are inconsistent then $[K^{\oplus}_{\alpha}]$ is simply $[K]$. The formulae in the new epistemic state can be determined by $K^{\oplus}_{\alpha} = th([K^{\oplus}_{\alpha}]) = th(f_{\mathcal{IS}}(\alpha))$.

We can now interpret the principle of minimal change with respect to this semantics. In fact, we can do so in a manner that integrates nicely with the sphere semantics for AGM revision. We consider ourselves to have a system of spheres centred on the most preferred worlds (i.e., those within the innermost dashed ring). When we perform abductive expansion of K with respect to α, we take those α-worlds closest to the most preferable worlds. We are, however, limited by the boundary $[K]$ and cannot move beyond this for expansion. For AGM revision, in the principal case where the new information α is inconsistent with the current epistemic state K, we move beyond the boundary $[K]$ in search of the closest α-worlds.

The following theorems show that the internal sphere semantics suggested here exactly characterises transitively relational partial meet abductive expansion functions.

Theorem 4.1 *Let $K \in \mathcal{K}$ be some belief set and $\mathcal{IS}$ any internal system of spheres in $\mathcal{M}_{\mathcal{L}}$ centred within $[K]$. If for any $\alpha \in \mathcal{L}$ we define $K^{\oplus}_{\alpha}$ to be $th(f_{\mathcal{IS}}(\alpha))$, then postulates* ($K^{\oplus}1$)—($K^{\oplus}8$) *are satisfied.*

Theorem 4.2 *Let $\oplus : \mathcal{K} \times \mathcal{L} \to \mathcal{K}$ be any function satisfying postulates* ($K^{\oplus}1$)—($K^{\oplus}8$). *Then for any belief set $K \in \mathcal{K}$ there is an internal system of spheres $\mathcal{IS}$ on $\mathcal{M}_{\mathcal{L}}$ centred within $[K]$ which, for all $\alpha \in \mathcal{L}$, satisfies $K^{\oplus}_{\alpha} = th(f_{\mathcal{IS}}(\alpha))$.*

In Figure 2 we continue the simple example outlined in the introduction. We consider a language consisting of only three propositions: r (rain), s (sprinkler) and g (wet_grass). Suppose we currently believe that if it rains then the grass gets wet and if the sprinkler is on, the grass gets wet. We are presented with the new information that the grass is wet (g). According to Figure 2 two worlds are preferred (shaded section), leading to the belief set $K^{\oplus}_{g} = Cn(\{s\} \cup \{g\})$; that is, the sprinkler was on (and the grass is wet).

5 Abductive Entrenchment

It has been shown that Grove's ordering on possible worlds is equivalent to an ordering over the formulae of $\mathcal{L}$ [7] (see also, [4] p. 95). Likewise, we can show that the internal system of spheres presented in the previous section also leads to an ordering over formulae.

In the AGM framework, an epistemic entrenchment is an ordering over the formulae of $\mathcal{L}$. Intuitively, it can be thought to represent a preference over formulae. In contraction, less entrenched formulae are given up in preference to more entrenched formulae. Such an ordering is defined as follows [5]:

Definition 5.1 *An ordering $\leq$ over $\mathcal{L}$ is an (AGM) epistemic entrenchment ordering if it satisfies the following conditions:*

(SEE1)	*For any $\alpha, \beta, \gamma \in \mathcal{L}$, if $\alpha \leq \beta$ and $\beta \leq \gamma$, then $\alpha \leq \gamma$*	*(transitivity)*
(SEE2)	*For any $\alpha, \beta \in \mathcal{L}$, if $\{\alpha\} \vdash \beta$, then $\alpha \leq \beta$*	*(dominance)*
(SEE3)	*For any $\alpha, \beta \in \mathcal{L}$, either $\alpha \leq \alpha \wedge \beta$ or $\beta \leq \alpha \wedge \beta$*	*(conjunctiveness)*
(SEE4)	*When $K \neq K_{\perp}$, $\alpha \notin K$ iff $\alpha \leq \beta$ for all $\beta \in \mathcal{L}$*	*(minimality)*
(SEE5)	*If $\beta \leq \alpha$ for all $\beta \in \mathcal{L}$, then $\vdash \alpha$*	*(maximality)*

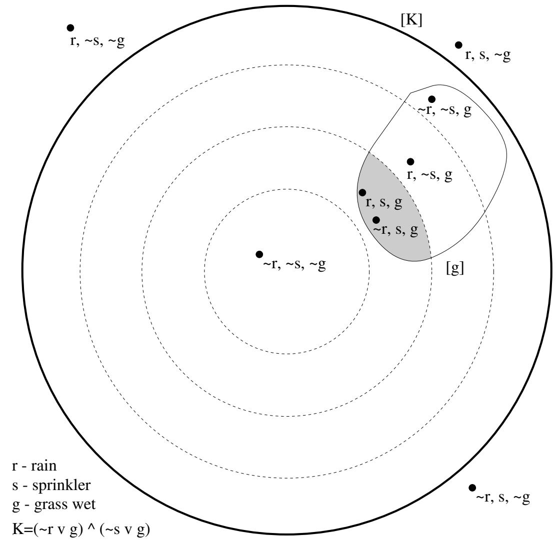

Figure 2: Example of an internal system of spheres.

We can view these conditions as specifying an ordering in which tautologies are maximally entrenched and non-beliefs are minimally entrenched. Therefore, the ordering is essentially one over the formulae in K.

Gärdenfors and Makinson [6] have also considered orderings satisfying (SEE1)—(SEE3). These are called expectation orderings. They used expectation orderings in an attempt to unify the area of belief revision with that of nonmonotonic inference.

In a similar manner, we can specify an ordering over formulae to help determine which formulae are to be included after abductive expansion. We define it as follows:

Definition 5.2 *An ordering $\leq$ over $\mathcal{L}$ is an abductive entrenchment ordering if it satisfies (SEE1)—(SEE3) and the following condition:*

(AE4) *When $K \neq K_\perp$, $\alpha \in K$ iff $\beta \leq \alpha$ for all β* *(maximality)*

This extra condition essentially specifies an ordering over formulae that are not believed. The result is an expectation ordering in which beliefs are maximally entrenched (AE4). We note the following properties of an abductive entrenchment ordering.

Lemma 5.1 *If $\leq$ satisfies* (SEE1)—(SEE3) *and* (AE4) *then it also has the following properties:*

(*i*) *If $\alpha \in K$ and $\beta \notin K$ then $\beta < \alpha$*

(*ii*) *When $K \neq K_\perp$, if $\neg\alpha \in K$ then $\alpha \leq \beta$ for all $\beta \in \mathcal{L}$*

(*iii*) *When $K \neq K_\perp$, if $K \cup \{\alpha\} \vdash \beta$, then $\alpha \leq \beta$*

(where $<$ refers to the strict part of $\leq$).

(*i*) says that beliefs are strictly maximally entrenched. (*ii*) tells us that disbeliefs (i.e., formulae inconsistent with K) are minimally entrenched. (*iii*) shows that a strengthened form of condition (SEE2) results from the addition of (AE4).

Intuitively, if $\alpha \leq \beta$, then it is no more difficult to assume β than α in an abduction. The following condition determines an abductive entrenchment ordering $\leq$ given an abductive expansion function $\oplus$ and a belief set K.

$$(C \leq) \qquad \alpha \leq \beta \textit{ iff either } \alpha \notin K^{\oplus}_{\neg\alpha\vee\neg\beta} \textit{ or } K \vdash \alpha \wedge \beta$$

This condition may be motivated in the following manner. Considering first the latter case, where both α and β are in K, being maximally entrenched (as they will be automatically included in the new epistemic state), they are both equally entrenched; hence, trivially $\alpha \leq \beta$. On the other hand, if $\alpha \notin K$, but $\beta \in K$, then surely $\alpha \leq \beta$. This can be obtained from condition (C$\leq$) due to the fact that $\alpha \notin K^{\oplus}_{\neg\alpha\vee\neg\beta}$ (so $\alpha \notin K$ by (K$^{\oplus}$3)). However, if neither α nor β is in K, the only thing we can claim is that not both α and β are in $K^{\oplus}_{\neg\alpha\vee\neg\beta}$ (one of α or β being in $K^{\oplus}_{\neg\alpha\vee\neg\beta}$ is a distinct possibility). In that case, it is clear that if $\alpha \notin K^{\oplus}_{\neg\alpha\vee\neg\beta}$, α cannot be strictly more entrenched than β. Hence, $\alpha \leq \beta$.

The next condition allows us to determine an abductive expansion function $\oplus$ for a particular K given an abductive entrenchment relation $\leq$.

$$(C\oplus) \qquad \beta \in K^{\oplus}_{\alpha} \textit{ iff either } \beta \in K \textit{ or both } \neg\alpha \notin K \textit{ and } \alpha \rightarrow \neg\beta < \alpha \rightarrow \beta$$

The principal part of this condition says that the new beliefs obtained by abductive expansion of K by α are the formulae β such that the information in β relative to α is easier to assume than that in $\neg\beta$ relative to α.

The following theorems show that the definitions above are in fact suitable. The first shows that an abductive expansion function for a particular K defined from an abductive entrenchment relation using condition (C$\oplus$) has the desired properties. The second shows the converse; that we can use an abductive expansion function $\oplus$ and condition (C$\leq$) to construct an abductive entrenchment relation with the desired properties.

Theorem 5.2 *Let $K \in \mathcal{K}$ be some belief set and $\leq$ an abductive entrenchment for K. If for any $\alpha \in \mathcal{L}$, we define $K^{\oplus}_{\alpha}$ using $(C\oplus)$, then* (K$^{\oplus}$1)—(K$^{\oplus}$8) *are satisfied as well as the condition* (C$\leq$).

Theorem 5.3 *Let $\oplus : \mathcal{K} \times \mathcal{L} \rightarrow \mathcal{K}$ be any function satisfying* (K$^{\oplus}$1)—(K$^{\oplus}$8). *Then, for any belief set $K \in \mathcal{K}$, if we define $\leq$ using* (C$\leq$) *then $\leq$ is an abductive entrenchment relation (i.e., it satisfies* (SEE1)—(SEE3) *and* (AE4)*) and condition $(C\oplus)$ is satisfied.*

In light of these results, we can view abductive entrenchment as an extension of epistemic entrenchment. In fact, both are different types of expectation ordering with epistemic entrenchment essentially ordering beliefs while abductive entrenchment orders non-beliefs. Therefore, given an expectation ordering, we can use that part of it which orders the elements of K to determine AGM contraction (and revision) and that part ordering the non-beliefs to perform abductive expansion. This is quite an interesting result in view of the suggested links between expectation orderings and nonmonotonic inference [6]. It shows what use may be made of two different "parts" of an expectation ordering for the purpose of belief change.

We defer an example of abductive entrenchment to the following section where we discuss how it may be used for default reasoning.

6 Default Reasoning

We have observed that abductive entrenchment is a special type of expectation ordering in which beliefs are maximally entrenched. Gärdenfors and Makinson [6] pointed out that expectation orderings can be used to deal with defaults but did not distinguish between that part of the expectation ordering over the beliefs and that part over the non-beliefs. They also identify these orderings with plausibility valuations. Williams [19] introduces *expectation rankings* — essentially a particular modelling of expectation orderings — and points out that those formulae ordered "below" the beliefs can be considered defaults; minimally entrenched formulae are neither beliefs nor defaults. Together with condition (C$\oplus$) this provides a very interesting way of viewing abductive entrenchment (or expectation orderings in general) and abductive expansion. If we wish to have a default of the form "whenever α is the case, then β is the case" we can achieve this by having $\alpha \rightarrow \neg\beta < \alpha \rightarrow \beta$ in the abductive entrenchment ordering according to condition (C$\oplus$). In this way, abductively expanding by α will ensure β is included in the new epistemic state[8].

Continuing our simple example, suppose our agent comes from an outback Australian town where it rarely rains. It might be more plausible for our agent to suppose, upon seeing that the grass is wet, that the sprinkler was on rather than it rained. This predisposition would be reflected in an abductive entrenchment including the following relative orderings: $g \rightarrow \neg s < g \rightarrow s$ and $g \rightarrow \neg r = g \rightarrow r$ (where $\alpha = \beta$ means $\alpha \leq \beta$ and $\beta \leq \alpha$). In this way, supposing $\neg g,\ r \notin K$, then $s \in K_g^{\oplus}$ and $r \notin K_g^{\oplus}$. Williams [19] achieves the same effect through the notion of a *cut* rather than condition (C$\oplus$) and Gärdenfors and Makinson [6] use the same inequality $\alpha \rightarrow \neg\beta < \alpha \rightarrow \beta$ over the entire expectation ordering.

Williams [19] further notes that "[u]sing results from Gärdenfors and Makinson [6] together with the observation that setting normal defaults to supernormal defaults by way of $\frac{\alpha:\beta}{\beta} \Longrightarrow \frac{\top:\alpha\rightarrow\beta}{\alpha\rightarrow\beta}$, it can be seen that expectation orderings are as expressive as normal defaults with linear priorities." One can then have a default like $\frac{\alpha:\beta}{\beta}$ (or $\frac{\top:\alpha\rightarrow\beta}{\alpha\rightarrow\beta}$) within abductive entrenchment by having $\alpha \rightarrow \neg\beta < \alpha \rightarrow \beta$. We can think of such a default in our abductive expansion framework as saying "if it is possible to incorporate α (into our current epistemic state) then include β too". Note that there is no need to check the consistency of β since, if $\neg\beta \in K$, then $\alpha \rightarrow \neg\beta \in K$ (i.e., $\neg\alpha \vee \neg\beta \in K$) and consequently $\alpha \rightarrow \neg\beta \geq \alpha \rightarrow \beta$. Thus condition (C$\oplus$) blocks the inclusion of β in the new epistemic state.

In light of this, we can suggest a simple way of viewing Poole's [17] system in terms of abductive expansion[9]. Poole's facts correspond to our K and the defaults are expressed in the manner outlined above. Poole's notion of explanation of α would in a certain sense correspond to abductive expansion of K by α. One difference, however, is that abductive expansion is not faced with the multiple extension problem since abductive entrenchment imposes restrictions that preclude this (and maintains consistency). Different extensions can be obtained by considering different abductive entrenchments. This correspondence is not entirely surprising given the abductive nature of Poole's explanation. It is also interesting to contrast this proposed view of Poole with that of Makinson and Gärdenfors [14] (also [6]) where a correspondence is shown between Poole's proposal and AGM full meet revision in which K_α^* is considered the revision of defaults (i.e., K represents the default set rather than the facts) by the new information.

Our view of the nature of the epistemic state K is slightly different from this view. We see K as containing those things that the agent truly believes. Things that the agent considers to be "usually" true etc., are not normally included in K but, rather, lie outside K and are ordered among the other non-beliefs. We consider this better than including the defaults within K since one "counterexample" would cause the removal of the default from K whereas, if the defaults are separate from K, there is no reason for a default to be "deactivated" in such a manner. On the other hand, Gärdenfors and Makinson [6] state: "[f]or so long as we are *using* a belief set K, its elements function as full beliefs. But as soon as we seek to *revise* K thus putting its elements into question, they lose the status of full belief and become merely expectations, ...". In our view K is always considered to consist of full beliefs. In summary, we believe that abductive expansion via abductive entrenchment lends itself to capturing an evolving epistemic state K (representing full beliefs) and defaults while Gärdenfors and Makinson's suggestion works well if we would like to determine which defaults (represented by K) to apply on a single occasion.

[8] We only consider the principal case where $\neg\alpha \notin K$.

[9] We consider Poole's proposal without his notion of constraints here, though it may be possible to capture this within abductive entrenchment also.

We can further illustrate this argument using the following fact about AGM revision

$$\text{If } \beta \in K^*_\alpha, \text{ then } \alpha \to \beta \in K.$$

Returning to our example, suppose $s \in K^*_g$. According to the fact above, we must have $g \to s \in K$ in order for this to be the case. That is, the agent must believe that "if the grass is wet, then the sprinkler was on" in order to come to believe that the sprinkler was on if it observed that the grass is wet. But this seems somewhat counterintuitive. In the case of abductive expansion, no such belief is required (nor would it have any effect).

7 Abductive Consequence Relation

Having noted some parallels between default reasoning and abductive expansion, it would be interesting to see what sort of (nonmonotonic) consequence relation results from the abductive expansion postulates. Adopting the approach Makinson and Gärdenfors [14] used for AGM revision, we translate our postulates for abductive expansion into conditions on a consequence relation $\mid\!\sim$ through the following definition.

Definition 7.1 $\alpha \mid\!\sim \beta$ *iff* $\beta \in K^\oplus_\alpha$.

We also adopt the notation $C(\alpha) = \{\beta \in \mathcal{L} : \alpha \mid\!\sim \beta\}$.

The first thing we need to decide is how to translate conditions of the form

$$\text{If } \alpha \in K, \text{ then } \ldots$$

When dealing with revision functions, such an antecedent would be translated in the following manner

$$\text{If } \top \mid\!\sim \alpha, \text{ then } \ldots$$

However, as noted in [14], this relies on the fact that $K^*_\top = K$; otherwise this translation would not work. In fact, this is not a consequence of our abductive expansion postulates. It would follow if we adopted the vacuity postulate (V) mentioned in Section 3.1. However, we have not chosen to include it among the abductive expansion postulates. We do, however, have the property that $K^\oplus_\perp = K$ by postulate ($K^\oplus 4$). Therefore, we would translate the above condition as

$$\text{If } \perp \mid\!\sim \alpha, \text{ then } \ldots$$

Moreover, this does not suffer a limitation noted in [14]. In that framework, since $K^*_\top \neq K$ when $K = K_\perp$, $\top \mid\!\sim \alpha$ works only if $K \neq K_\perp$. In our case $K^\oplus_\perp = K$ even when $K = K_\perp$; so $\perp \mid\!\sim \alpha$ works unconditionally. This consideration motivates the following translation of the abductive expansion postulates ($K^\oplus 1$) — ($K^\oplus 8$).

($\mid\!\sim$1) $C(\alpha) = Cn(C(\alpha))$
($\mid\!\sim$2) If $\perp \not\mid\!\sim \neg\alpha$, then $\alpha \mid\!\sim \alpha$
($\mid\!\sim$3) If $\perp \mid\!\sim \beta$, then $\alpha \mid\!\sim \beta$
($\mid\!\sim$4) If $\perp \mid\!\sim \neg\alpha$, then $\perp \mid\!\sim \beta$ iff $\alpha \mid\!\sim \beta$
($\mid\!\sim$5) If $\perp \not\mid\!\sim \neg\alpha$, then $\alpha \not\mid\!\sim \neg\alpha$
($\mid\!\sim$6) If $\perp \mid\!\sim \alpha \leftrightarrow \beta$, then $\alpha \mid\!\sim \gamma$ iff $\beta \mid\!\sim \gamma$
($\mid\!\sim$7) If $\alpha \mid\!\sim \gamma$, then $\alpha \vee \beta \mid\!\sim \alpha \to \gamma$
($\mid\!\sim$8) If $\alpha \vee \beta \not\mid\!\sim \neg\alpha$, then $\alpha \vee \beta \mid\!\sim \gamma$ implies $\alpha \mid\!\sim \gamma$

These properties can now be analysed in the manner of Makinson and Gärdenfors [14] and Kraus, Lehmann and Magidor [12]. We defer a detailed treatment at this time and simply make some brief observations. However, in light of the fact that abductive entrenchment is a type of expectation ordering and results by Gärdenfors and Makinson [6] relating expectation orderings to properties on consequence relations, we expect abductive expansion to satisfy similar properties to those noted there.

The principle of monotonicity clearly does not hold of abductive expansion; nor does it hold for the conditions above. So $\mid\!\sim$ is a nonmonotonic consequence relation. Condition ($\mid\!\sim$1) (as noted in [14]) is a common principle holding in many nonmonotonic systems. ($\mid\!\sim$2) is a restricted version of (Reflexivity)[10]. In contrast to AGM revision we can see that reflexivity does not always hold. ($\mid\!\sim$3) follows from the nonmonotonic principle (And). Condition ($\mid\!\sim$4) is a boundary condition which does not appear to have an analogue among the common properties of consequence relations. ($\mid\!\sim$5) says that $\mid\!\sim$ does not introduce inconsistency. It should not be confused with (Consistency Preservation). ($\mid\!\sim$6) is derived from a postulate which is a stronger version of extensionality whose translation holds for many nonmonotonic systems. ($\mid\!\sim$7) is a consequence of (Conditionalisation). Condition ($\mid\!\sim$8) follows from (Rational Monotony).

8 Related Work

A belief change operation known as *deliberate expansion*, which involves abductive inference, has been discussed by Levi [13]. However, he does not present rationality postulates nor entirely embrace the AGM view of belief revision. Kakas and Mancarella [11] suggest a method for performing database updates which is very similar to that proposed here. They claim that such an update makes information explicit that might otherwise remain implicit and could be lost. Furthermore, if a knowledge base has been structured in a certain way (with respect to certain propositions or predicates, for instance), then this method may be able to maintain that structure.

Although the basic idea behind the work presented here and that of Kakas and Mancarella is quite close, the underlying frameworks are somewhat different. They consider a deductive database separated into intensional and extensional parts. Furthermore, a deductive database is considered to be a logic program. In contrast, the framework presented here is intended to be quite general and theoretical in nature. As such, many considerations are highly idealised. Abductive approaches to logic programming (see [10] for an overview) tend to be more pragmatic as they impose more restrictions on the underlying framework. Such restrictions will lead to constraints on the possible abductive expansion functions (i.e., reduction of permissible systems of spheres, abductive entrenchment, etc.). A closer correspondence is likely to be obtained by investigating belief base approaches to belief revision rather than the belief set approach adopted here. However, as with the AGM, the framework presented here may not only provide valuable insight into such approaches but stand as a theoretical extreme to which such approaches would converge.

Boutilier and Becher [3] and Williams *et al.* [20] investigate notions of explanation based on belief revision. In contrast to the method proposed here, where we attempt to incorporate an explanation, they attempt to use revision to determine explanations[11].

An alternative investigation of the relationship between abduction and belief revision is that of Aravindan and Dung [2]. They examine the use of abduction in belief (base) contraction.

9 Conclusions and Future Work

We have elaborated a framework for an abductive belief expansion operator introduced by Pagnucco *et al.* [15] providing a simplified version of their postulates and introducing constructions based on systems of spheres and abductive entrenchment. This operation was proposed as a model for the claim that agents first seek an explanation or justification for new information and incorporate

[10] Consult Gärdenfors and Makinson [6] and Kraus, Lehmann and Magidor [12] for a discussion of this principle and those that follow.

[11] More precisely, we attempt to incorporate an abduction. We are not claiming that our definition of abduction exactly characterises the notion of explanation but have used the term here to highlight the difference in the respective approaches.

this into their epistemic state along with the new information rather than just incorporating the new information alone. The sphere semantics presented is analogous to that proposed by Grove [7] for the AGM framework. Similarly, the abductive entrenchment construction is an analogue of epistemic entrenchment developed by Gärdenfors and Makinson [5]. Both can in fact be considered extensions of this earlier work.

Abductive expansion represents a rejection of the AGM notion of minimal change for belief expansion which is taken with respect to set inclusion. However, we have shown how the principle of minimal change can be interpreted in terms of the sphere semantics proposed here. This is consistent with the way in which the principle of minimal change is viewed for the operations of contraction and revision. Moreover, the abductive entrenchment construction can be considered an extension to that of epistemic entrenchment. While epistemic entrenchment orders the agent's beliefs, abductive entrenchment essentially orders those formulae for which the agent has no opinion. It provides an insight into what use may be made of different parts of an expectation ordering. Using observations made by Gärdenfors and Makinson [6] and Williams [19] we have suggested how default reasoning can be performed and viewed in terms of abductive expansion. We believe the interpretation in terms of abductive expansion provides a more natural view than one in terms of AGM revision. This would hold equally well for an abductive revision operator (see below). A translation of the abductive expansion postulates into conditions on a nonmonotonic consequence relation $\mid\!\sim$ along the lines of Makinson and Gärdenfors [14] was also performed but needs to be investigated more closely.

Having investigated an abductive belief expansion operator $\oplus$ it would be interesting to take the AGM approach and define an abductive revision operator $\otimes$ using the Levi identity [4]: $K^{\otimes}_{\alpha} = (K^{\ominus}_{\neg\alpha})^{\oplus}_{\alpha}$. Here $K^{\ominus}_{\neg\alpha}$ represents any type of contraction. The resulting revision operator would allow incorporation of new information that is not consistent with the current epistemic state. Leaving aside issues related to iterated belief change which may need to be considered, and taking $\ominus$ to be AGM contraction, early indications are that in the principal case, where the epistemic input is self-consistent but not consistent with K, the class of abductive revision functions would be identical to the class of AGM revision functions. In the situation where the epistemic input is inconsistent in itself we are left with the current epistemic state (success is not guaranteed in this case). Otherwise, if the epistemic input is consistent with K, the abductive revision collapses to abductive expansion.

Acknowledgements

The authors would like to thank Pavlos Peppas, Mary-Anne Williams and the anonymous referees for their insightful comments and suggestions. Comments from our colleagues in the Knowledge Systems Group have also helped to improve the content and presentation of this paper. The first author is supported by an Australian Postgraduate Research Award.

References

[1] C. E. Alchourrón, P. Gärdenfors, and D. Makinson. On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic*, 50:510–530, 1985.

[2] C. Aravindan and P. M. Dung. Belief dynamics, abduction, and databases. In C. MacNish, D. Pearce, and L. Pereira, editors, *Proceedings of the Fourth European Workshop on Logics in Artificial Intelligence (JELIA'94)*, pages 66–85, 1994.

[3] C. Boutilier and V. Becher. Abduction as belief revision. *Artificial Intelligence*, 1995. To appear.

[4] P. Gärdenfors. *Knowledge in Flux: Modeling the Dynamics of Epistemic States*. Bradford Books, MIT Press, Cambridge Massachusetts, 1988.

[5] P. Gärdenfors and D. Makinson. Revisions of knowledge systems using epistemic entrenchment. In *Proceedings of the Second Conference on Theoretical Aspect of Reasoning About Knowledge*, pages 83–96, 1988.

[6] P. Gärdenfors and D. Makinson. Nonmonotonic inference based on expectations. *Artificial Intelligence*, 65:197–245, 1994.

[7] A. Grove. Two modellings for theory change. *Journal of Philosophical Logic*, 17:157–170, 1988.

[8] G. H. Harman. *Change in View: Principles of Reasoning*. MIT Press, 1986.

[9] P. Jackson. Propositional abductive logic. In *Proceedings of the Seventh Conference of the Society for the Study of Artificial Intelligence and Simulation of Behaviour*, pages 89–94, 1989.

[10] A. C. Kakas, R. A. Kowalski, and F. Toni. Abductive logic programming. *Journal of Logic and Computation*, 2(6):719–770, 1993.

[11] A. C. Kakas and P. Mancarella. Database updates through abduction. In *Proceedings of the Sixteenth Conference on Very Large Databases*, pages 650–661. Brisbane, Australia, 1990.

[12] S. Kraus, D. Lehmann, and M. Magidor. Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial intelligence*, 44:167–207, 1991.

[13] I. Levi. *The Fixation of Belief and its Undoing*. Cambridge University Press, 1991.

[14] D. Makinson and P. Gärdenfors. Relations between the logic of theory change and nonmonotonic logic. In A. Fuhrmann and M. Morreau, editors, *The Logic Of Theory Change*, pages 185–205. Lecture Notes in Artificial Intelligence 465, Springer Verlag, 1990.

[15] M. Pagnucco, A. C. Nayak, and N. Y. Foo. Abductive expansion: The application of abductive inference to the process of belief change. In C. Zhang, J. Debenham, and D. Lukose, editors, *Proceedings of the Seventh Australian Joint Conference on Artificial Intelligence*, pages 267–274, Armidale, Australia, 1994.

[16] G. Paul. Approaches to abductive reasoning: An overview. *Artificial Intelligence Review*, 7:109–152, 1993.

[17] D. Poole. A logical framework for default reasoning. *Artificial Intelligence*, 36:27–47, 1988.

[18] M. Ranney and P. Thagard. Explanatory coherence and belief revision in naive physics. In *Proceedings of the Tenth Annual Conference of the Cognitive Science Society*, pages 426–432. Earlbaum, 1988.

[19] M.-A. Williams. Changing nonmonotonic inference relations: A computational model. In *Proceedings of the Second World Conference on Foundations for Artificial Intelligence*. Paris, 1995. to appear.

[20] M.-A. Williams, M. Pagnucco, N. Y. Foo, and B. Sims. Determining explanations using knowledge transmutations. In *Proceedings of the Fourteenth International Joint Conference on Artificial Intelligence*. Montreal, 1995. to appear.