The Relationship Between Abduction and Changes in Belief States*

Maurice Pagnucco
email : morri@cs.su.oz.au

Norman Foo
email : norman@cs.su.oz.au

Knowledge Systems Group
Basser Department of Computer Science
Madsen Building F09
University of Sydney
NSW, 2006, Australia.
fax : +61-2-692-3838

Abstract

Belief revision attempts to model how an agent's belief states change under the influence of new information. The AGM framework, for example, identifies three types of change: expansion, contraction and revision. But, given an updated belief state (i.e., one that has been expanded, contracted or revised) and the nature of the new information, is it possible to determine or say something about the initial belief state? In this paper we consider contraction and argue that the process of attempting to determine the nature of an initial belief state from a contracted belief state and an epistemic input can be considered one of abduction. We also briefly look at the implications of this in terms of "sphere" semantics.

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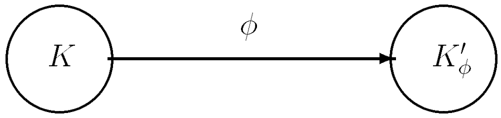
1 Introduction

Over the past number of years increasing attention has been focused on the area of belief revision — an attempt to model how an agent's beliefs (epistemic states) change when presented with new information (an epistemic input). This is commonly represented as a transition from an initial belief state K to an updated belief state K'_ϕ with respect to an epistemic input ϕ.



If we are using some type of logic to model this process then belief states K, K'_ϕ and the epistemic input ϕ are usually represented by sets of formulae and a single formula respectively. In this case, a further important restriction is that, where possible, the belief states should remain (logically) consistent.

In the framework developed by Alchourrón, Gärdenfors and Makinson [1, 2] (henceforth referred to as the AGM framework), which we shall be using here, three types of change or transition are identified:

expansion (K^+_ϕ) straightforward addition of a new belief ϕ and its logical consequences to the belief state K;

contraction (K^-_ϕ) removal of a belief ϕ from a belief state K;

revision (K^*_ϕ) addition of a new belief ϕ which may contradict currently existing beliefs in the belief state K.

So, in the situation outlined above, the new belief state K'_ϕ will be represented by either K^+_ϕ, K^-_ϕ or K^*_ϕ. But, consider the situation where the resulting belief state K'_ϕ is already known together with the epistemic input ϕ. Is it possible to determine the initial belief state K or at least say something about it? It is the contention of this paper that we can in fact say something about the initial belief state even though we may not be able to determine it exactly[1]. In particular, we shall concern ourselves with the case

[1] Even in moving from K to K'_ϕ in the AGM framework, the resulting belief state K'_ϕ is not necessarily uniquely determined.

of contraction — from a contracted belief state K_ϕ^- and epistemic input ϕ we shall attempt to say something about the initial belief state K. We shall argue that this process may be viewed as a characterisation of some notion of abduction, which we shall present in one of the following sections. Briefly, the argument proceeds as follows. The components of an abduction are

(i) a belief state κ and an observation ϕ such that $\phi \notin \kappa$, and

(ii) a process that derives from κ and ϕ a new belief state K such that $\phi \in K$.

Our key contention is that κ can be identified with K_ϕ^-, i.e., the abduced belief state K is viewed as the restoration of some missing beliefs that can account for the observation ϕ.

The significance of this view is that we can consider abduction as a process that attempts to take us from a contracted belief state and an epistemic input to the original (or uncontracted) belief state. In other words, we begin with a belief state from which beliefs have been removed and we are trying to determine the nature of the initial belief state.

In the following section we outline the nature of the language and the logic assumed in this paper. Section 3 presents the notion of abduction that we shall be using here. In scetion 4 we briefly present the AGM postulates for contraction while in section 5 we argue that, under certain circumstances, abduction corresponds to an attempt to determine an initial belief state from an epistemic input and a contracted belief state. Our conclusions are given in section 6.

2 Preliminaries

We shall consider a fixed language $\mathcal{L}$ based on propositional logic. $\mathcal{L}$ will consist of the standard connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$ and the symbol $\perp$ (falsum). The symbols ϕ, ψ will be used to range over formulae in $\mathcal{L}$ and Γ, Δ over sets of formulae. The underlying logic considered will be assumed to include classical propositional calculus. We shall also suppose that this logic is compact (i.e., if $\Gamma \models \phi$ then $\Gamma' \models \phi$ for some finite $\Gamma' \subseteq \Gamma$), that it contains the inference rule of modus ponens and, for the AGM postulates, satisfies the rule of introduction of disjunction into the premises (i.e., $\Gamma \cup \{\phi_1 \vee \phi_2\} \models \psi$ whenever $\Gamma \cup \{\phi_1\} \models \psi$ and $\Gamma \cup \{\phi_2\} \models \psi$).

3 Abduction

In this section we shall briefly introduce the notion of abduction which we are adopting here. We begin with a fairly standard definition of abduction. Similar definitions may be found in [5, 6, 8].

Definition 3.1 *(Abduction)*
An abduction of a formula ϕ with respect to a domain theory Γ (a set of formulae) is a set of formulae E such that:

(i) $\Gamma \cup E \models \phi$*;*

(ii) $\Gamma \cup E$ *is consistent (i.e.,* $\Gamma \cup E \not\models \bot$*) .*

We also say that E is abduced from Γ and ϕ.

Note that unlike some other work in this area, we have avoided calling this notion *explanation* since we are not necessarily considering an explanatory interpretation of the results. This definition could also be extended to consider abductions of a set of formulae Δ instead of a single formula ϕ but we wish only to concentrate on the single formula case here.

Utilising this definition we can make the following observation that is an immediate consequence of compactness

Observation 3.1 *If an abduction E of a formula ϕ with respect to a domain theory Γ exists then a finite abduction $E' \subseteq E$ (where E' is understood to be finite) of ϕ with respect to Γ exists.*

Once we realise that if we can obtain an abduction then we can obtain a finite abduction it is again straightforward to show

Lemma 3.2 *If a finite abduction E of ϕ with respect to Γ exists then it can be represented by a single formula ψ (i.e.,* $\Gamma \cup \{\psi\} \models \phi$*,* $\Gamma \cup \{\psi\} \not\models \bot$*).*

proof
By the transitivity of $\models$.

Given a formula ϕ and domain theory Γ we can usually derive a number of abductions. The ones that we are interested in are the *minimal abductions*, defined in the following way:

Definition 3.2 *(minimal abduction)*
We say that an abduction ψ of ϕ with respect to Γ is a minimal abduction iff for any ψ', if $\psi \models \psi'$ and ψ' is also an abduction of ϕ with respect to Γ then $\models \psi \leftrightarrow \psi'$ (where ψ, ψ' and ϕ are single formulae).

Note that for minimal abduction we use abductions characterised by a single formula ψ instead of a set of formulae E which would have been fine but we wish to stress that we are considering finite abductions. In fact, it is quite easy to show that any minimal abduction in the language adopted here is finite in nature. We could have adopted this single formula approach from the beginning but we wanted to show that it is related to the more common approach of defining an abduction (or explanation) as a set of formulae. For example, if $\Gamma = \{a \to b, c \to b, d \to e\}$, $\phi = b$ then possibilities for ψ include a, c (either is minimal) but $a \wedge d$ is not minimal. Other notions of minimality (or simplicity) can be found in [5, 6]. The reason for adopting some form of minimality criteria is to exclude superfluous abductions.

4 AGM Postulates

The AGM postulates for expansion, contraction and revision attempt to characterise classes of functions for modelling the manner in which a rational agent alters its beliefs. A common manifestation of this framework represents belief states in terms of belief sets (deductively closed sets of formulae) and epistemic inputs in terms of a single formula ϕ. The language used is similar to that presented in section 2.

In this presentation we are mainly concerned with the process of contraction so, for completeness, we reproduce the six basic contraction postulates [2]:

(K^-1)	K^-_ϕ is a belief set	(closure)
(K^-2)	$K^-_\phi \subseteq K$	(inclusion)
(K^-3)	If $\phi \notin K$, then $K^-_\phi = K$	(vacuity)
(K^-4)	If not $\models \phi$, then $\phi \notin K^-_\phi$	(success)
(K^-5)	If $\phi \in K$, then $K \subseteq (K^-_\phi)^+_\phi$	(recovery)
(K^-6)	If $\models \phi \leftrightarrow \psi$, then $K^-_\phi = K^-_\psi$	(preservation)

For the process of expansion it is sufficient to note that it can be fully characterised in the following manner

$$K^+_\phi = Cn(K \cup \{\phi\}) \tag{1}$$

(where Cn represents the classical propositional consequence operation — i.e., $Cn(\Gamma) = \{\phi : \Gamma \models \phi\}$). The process of revision may be derived from that of contraction through the use of the so-called Levi Identity $K^*_\phi = (K^-_{\neg\phi})^+_\phi$. The interested reader is referred to [1, 2] for a more thorough treatment of this material.

5 Abduction and Contraction

Recall, from the introduction, that we are attempting to determine an initial belief state K given a contracted belief state K^-_ϕ and the epistemic input ϕ where K and K^-_ϕ are belief sets (i.e., deductively closed). We shall not consider all cases here since, apart from not having enough space, some yield less interesting results — if we had $\phi \notin K$ then, by (K^-3), $K = K^-_\phi$ and if $\models \phi$ (i.e., ϕ is a theorem of our logic) then it could not have been removed from K (i.e., it is still in K^-_ϕ). Furthermore, we shall assume that we are attempting to determine a logically consistent belief set K. So, we are looking at the situation where $\not\models \phi$ and determining a consistent K which contains ϕ (reasonable assumptions under the given conditions!).

5.1 Recovery

Now, from the postulate of recovery (K^-5) and the remaining postulates of contraction together with those for expansion it can be shown that, if $\phi \in K$ (the case we are investigating) then $K = (K^-_\phi)^+_\phi$ (see [2]). This tells us that if we wish to recover K from K^-_ϕ all we need do is expand K^-_ϕ by ϕ (i.e., add ϕ to K^-_ϕ and take its closure)!

The postulate of recovery is a rather contentious one and has generated some controversy concerning its validity [7, 4]. Let us suppose that some other formula[2] ψ may be added to K^-_ϕ to recover K

$$K = (K^-_\phi)^+_\psi \text{ (assuming } \phi \in K) \tag{2}$$

[2]We shall consider ψ to be a single formula but it makes little difference if we consider adding a set of formulae since we can prove a similar result to Lemma 3.2.

We note that recovery is a special case of this formula, when ψ is ϕ.

This alternative to recovery is similar to that proposed by Hansson [4]. He in fact defines a notion called *core-retainment* which asserts that if $x \in K$ and $x \notin K^-_p$ then there exists a set $A \subset K$ such that $p \notin Cn(A)$ and that $p \in Cn(A \cup \{x\})$[3]. In our sitation, ϕ corresponds to p while ψ corresponds to x. Whereas Hansson is interested in the set A as a characterisation of contraction, we are interested in the formula ψ (or x in the above definition). Interestingly enough, Hansson proves ([4] Theorem T1) that if an operation on a belief set satisfies (K^-1) - (K^-4), (K^-6) and core-retainment then it also satisfies recovery. This suggests that there may be alternatives to recovering the initial belief state from a contracted one but that the recovery property will still be satisfied when we are dealing with belief sets (this may not be the case if we represent a belief state using a non-closed belief base). Therefore, our formulation of recovery in (2) is plausible but, when we are dealing with belief sets, we could simply add the contracted formula to recover the initial belief state.

From this, we can use our representation for expansion (1) to show that $K^-_\phi \cup \{\psi\} \models K$. In particular, $K^-_\phi \cup \{\psi\} \models \phi$ since $\phi \in K$. Now, provided that ψ is consistent with K^-_ϕ($i.e.$, $\neg\psi \notin K^-_\phi$ or $K^-_\phi \cup \{\psi\} \not\models \bot$) then this corresponds exactly to our definition of an abduction of ϕ with respect to K^-_ϕ (with Lemma 3.2). Ensuring that ψ is consistent with K^-_ϕ will also ensure that the belief set K, determined in this manner, is also consistent (which is what we desire). So, to recover the initial belief state in this way we can use the process of abduction to suggest some formula ψ to add to our contracted belief state. The case represented by the recovery postulate is in fact the trivial abduction where we just use ϕ to account for itself. We could also show that expanding K^-_ϕ by an abduction would give us a belief set containing ϕ. Minimal abductions correspond to a notion of adding the "weakest" (in the sense of entailment) formula to recover K.

5.2 Sphere Semantics

Grove [3] presents a "sphere" semantics for expansion, contraction and revision functions in the AGM framework. He considers the set of all maximal consistent extensions of $\mathcal{L}$ which can be thought of as the set of possible

[3] We have retained Hansson's notation.

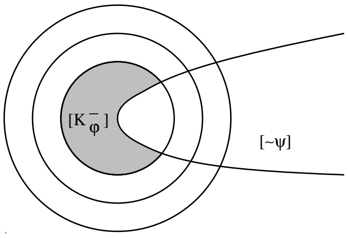


Figure 1: System of spheres showing $[K_\phi^-] - [\neg\psi]$.

worlds. In this approach, a belief set K is represented by a subset of this set of maximal consistent extensions with each extension in the subset containing the sentences in K. This subset is denoted $[K]$ and may be thought of as the set of possible worlds consistent with K. A similar definition may be used for any formula ϕ to determine the set of possible worlds $[\phi]$ consistent with ϕ (i.e., that include ϕ). Grove also defines the notion of a *system of spheres centred on* $[K]$ as a collection of subsets of the set of maximally consistent extensions satisfying a number of conditions (the interested reader us referred to [3] or [2] pp. 83 - 86).

We note that, within Grove's sphere semantics for the AGM postulates, contraction may be characterised by the formula $[K_\phi^-] = [K] \cup C(\neg\phi)$ where $C(\neg\phi)$ the set of possible worlds containing $\neg\phi$ that are closest to $[K]$. In the situation outlined above, we can show that $[K] \supseteq [K_\phi^-] - [\neg\psi]$. That is, to determine a set of possible worlds that might characterise the initial belief set K we remove the worlds containing $\neg\psi$ (where ψ is our abduction) from those of $[K_\phi^-]$. Figure 1 shows a representation of a system of spheres with the shaded part indicating $[K_\phi^-] - [\neg\psi]$.

Minimal abductions would tend to reduce the number of possible worlds eliminated. So, in this sense abduction corresponds to a mechanism for reducing the number of possible worlds available for consideration. This is as we might expect since abduction, in a certain sense, makes hypotheses about a domain thus effectively reducing uncertainty.

6 Conclusions

We have argued that our notion of abduction corresponds to an attempt to determine an initial belief state (formulated within the AGM framework) from a contracted belief state and an epistemic input under certain conditions. We have also argued a connection with a possible worlds semantics for the AGM postulates. The advantage of this approach is that we have established a possible connection between abduction and an area which has been intensively investigated, i.e., belief revision (with regards the AGM framework).

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