CSC C63 Midterm Exam

Mar 2, 2015

1 hour and 50 minutes

NAME:

Calculators are not permitted (nor would they be useful).

This is a closed book exam.

Ask an invigilator if there is anything that you do not understand completely.

1. **(16 pts) Short Answers.**

   (a) **(4 pts)** What does it mean for a set to be countable?

   (b) **(5 pts)** Define the set of problems P.

   (c) **(7 pts)** (i) State Hilbert's 10th Problem. (ii) What is the status of this problem?

2. **(8 pts)** This figure represents a Turing Machine at some point during its run.

(a) **(4 pts)** Give the **configuration** that represents this figure.

(b) **(4 pts)** Part of the transition function says:

$$(q_4, 1) \rightarrow (q_2, x, \text{Left})$$

Give the **configuration** representing the Turing Machine after the next step.

3. **(30 pts)** Consider the following language:

$A = \{(< P >, k) :$ there are at least $2k$ integers x such that
at least one of $P(x)$ or $P(x+1)$ halts and returns a number larger than $x\}$

P is a Turing Machine with non-negative integers as inputs and outputs; k is a non-negative integer.

(a) **(15 pts)** Is A decidable? Prove your answer.

(b) **(3 pts)** State the language $\overline{A}$.

(c) **(12 pts)** Either A or $\overline{A}$ is recognizable. Which one? Prove that it is recognizable.

4. **(6 pts)** You are given an integer x and you wish to count how many numbers divide evenly into x, including 1 and x. You use the following algorithm:

Input: x
Count := 0
for $i = 1$ to x
    test whether i divides evenly into x
    if i divides evenly into x then Count:=Count+1
return Count

Is this a polytime algorithm? Explain your answer.

5. **(8 pts)**

Prove that the following problem is in NP.

DENSE-SUBGRAPH
Input: A graph G.
Question: Is there a subset of vertices S in G such that the number of edges with both endpoints in S is at least $3|S|$?

6. **(9 pts)** A, B are enumerable languages. Prove that $A \cap B$ is enumerable.