CSC C63 Midterm Exam

Mar 12, 2014

1 hour and 50 minutes

NAME:

Calculators are not permitted (nor would they be useful).

This is a closed book exam.

Ask an invigilator if there is anything that you do not understand completely.
1. (12 pts) Short Answers.

(a) (4 pts) Name any NP-complete problem.

(b) (4 pts) State Church’s Thesis.

(c) (4 pts) What does it mean for a verification algorithm to be polynomial time?
2. (10 pts) This figure represents a Turing Machine at some point during its run.

(a) (5 pts) Give the configuration that represents this figure.

(b) (5 pts) Part of the transition function says:

\[(q_3, y) \rightarrow (q_5, t, \text{Left})\]

Draw a figure representing the Turing Machine after the next step.
3. (30 pts) Consider the following language:

\[
A = \{ (\langle P \rangle, k) : \text{there are at least } k \text{ integers } x \text{ such that } P(x) \text{ halts and returns a number that is at least } xk + 5 \}
\]

\(P\) is a Turing Machine with non-negative integers as inputs and outputs; \(k\) is a non-negative integer.

(a) (15 pts) Is \(A\) decidable? Prove your answer.
(b) \((15 \text{ pts})\) Either \(A\) or \(\overline{A}\) is recognizable. Which one? Prove that it is recognizable.
4. (24 pts) Consider the following problems.

**COST-OF-MAX-PATH**

**Input:** A connected graph $G$, with an integer weight $w_e \geq 0$ on each edge $e$, and two specified vertices $u, v$.

**Output:** The total cost of the path in $G$ from $u$ to $v$ with the greatest total weight.

Note: a path cannot repeat any vertices. So the maximum weight path can only visit each vertex at most once.

**DCMP**

**Input:** A connected graph $G$, with an integer weight $w_e \geq 0$ on each edge $e$, two specified vertices $u, v$, and a target $T$.

**Question:** Is there a path in $G$ from $u$ to $v$ of total weight at least $T$?

DCMP stands for Decision-Cost-of-Max-Path.

(a) (6 pts) Prove that DCMP is in NP.
(b)  **(8 pts)** Show that if there is a polytime algorithm for DCMP then there is a polytime algorithm for COST-OF-MAX-PATH.
(c) (10 pts) Recall the problem:

**HAM-PATH**

**Input:** A connected graph $G$ and two specified vertices $u, v$.

**Question:** Is there a Hamilton path in $G$ from $u$ to $v$?

Recall that a Hamilton path visits every vertex in $G$ exactly once. (Usually we don’t specify that $G$ is connected, but it is convenient to do so here, and it makes little difference since the condition is easy to check.)

Prove that $\text{DCMP} \geq_P \text{HAM-PATH}$
5. (8 pts) \( A, B, C \) are recognizable languages. Prove that \( A \cup B \cup C \) is recognizable.