CSC C63  Spring 2016, Assignment #4

Due: Fri April 1 at 3:00 in the course dropbox.

This assignment has 2 pages

For this assignment, you may work with one partner. Put your partner’s name on the front page of your assignment. You and your partner may discuss the problems, and solve them together. But you must write your own solutions without looking at your partners written solution. Do not take any written notes away from meetings with your partner.

It would be helpful if you and your partner placed your assignments in the dropbox consecutively.

Do not discuss the problems with anyone other than: your partner; the instructor; the TA. Do not use any materials that you find on the internet. Do not use any old solution sets. Read the plagiarism section on the course web page before beginning the assignment.

You can assume as fact that the following problems are NP-complete: SAT, 3-SAT, IND-SET, CLIQUE, VERTEX-COVER, SUBSET-SUM, HAMPATH (both the directed and undirected versions).

In one problem, you are asked to give a good reason for your answer. Your mark will be based on the strength of the reason. Saying something like "otherwise, P=NP" is a very strong reason. Saying something like "I think it’s NP-complete because it looks a bit like CLIQUE" is a weak reason.

If you present an algorithm (Turing Machine) which runs in logspace or polyspace, be sure to include an analysis of the space that it uses.

1. (20 pts) Consider the following problem:

   **3PRIMES**
   
   **Input:** An integer \( n \).
   
   **Question:** Is \( n \) the product of three primes?
   
   Eg. the answer is YES for 102 = 2 × 3 × 17 and 75 = 3 × 5 × 5, but NO for 21 = 3 × 7 and 150 = 2 × 3 × 5 × 5.

   (a) Prove that 3PRIMES is in NP\( \cap \)co-NP
   
   (b) Do you think 3PRIMES is NP-complete? Give a good reason for your answer.

2. (20 pts) Prove that NP\( \subseteq \)PSPACE by using the NDTM definition of NP. Specifically: Suppose that a language \( L \) is decided by a polynomial time Nondeterministic Turing Machine \( M \). Prove that there is a polynomial space Turing Machine \( R \) which takes an input string \( I \) and determines whether it is possible for \( M(I) \) to halt accept. Then explain that \( R \) decides \( L \).

3. (15 pts)

   **MOREHAMPATHS**
   
   **Input:** A graph \( G \) and four specified vertices, \( u_1, v_1, u_2, v_2 \).
   
   **Question:** Does \( G \) have more Hamilton paths from \( u_1 \) to \( v_1 \) than from \( u_2 \) to \( v_2 \)?

   Prove that MOREHAMPATHS \( \in \) PSPACE.
4. (25 pts)

(a) In this problem, you need to verify the answer to a multiplication.

**ADDITION**

**Input:** Three integers $a, b, c$ presented in binary

**Question:** Is $a + b = c$?

Prove that ADDITION is in $L$.

*Clarification:* $a, b, c$ are written in binary on the input tape. They are separated by a hash-character. Note that the size of the input is the total number of digits in $a, b, c$. So for your algorithm to run in $L$, the space used must be a constant multiple of the logarithm of the number of digits.

(b) **SUM-TO-PALINDROME**

**Input:** Two integers $a, b$ presented in binary

**Question:** Is $a + b$ a palindrome?

Prove that SUM-TO-PALINDROME is in $L$. Again, $a, b$ are written in binary on the input tape and are separated by a hash-character.

5. (20 pts) A directed graph is said to be strongly connected if for every pair of vertices $u, v$ there is both a path from $u$ to $v$ and a path from $v$ to $u$.

**STRONGLY-CONNECTED**

**Input:** A directed graph $G$.

**Question:** Is $G$ strongly connected?

Prove that STRONGLY-CONNECTED is in NL.