CSC 2427/MAT 1500 Winter 2025, Assignment #3

Due: Mon, Apr 7 at 11:59 PM. Emailed to molloy@cs.toronto.edu

I will not answer any questions about the assignment after Apr 5.

You may consult the Alon-Spencer and Molloy-Reed books. You may also consult an introductory graph theory textbook, such as the one by West. You may not consult any other materials, including online sources.

You must solve all problems on your own. Do not consult with anyone other than the instructor.

You must write complete proofs. We will grade a subset of the problems.

1. **15 pts** H is an r-uniform hypergraph, $r \geq 3$. Every vertex lies in at most Δ hyperedges.
   (a) Use the version of the Local Lemma with condition $p(d+1) < \frac{1}{e}$ to prove that H can be q-coloured with $q = (er\Delta)^{\frac{1}{r-1}}$.
   (b) Use the cluster expansion version of the Local Lemma to prove that H can be q-coloured for $q = c\Delta^{\frac{1}{r-1}}$ with a smaller constant c.

2. **20 pts** Problem 8 from Chapter 5 of Alon & Spencer. In other words: We saw in class that we can always form a non-repetitive sequence from any lists of size 5. Improve this to lists of size 4, using a hint.

3. **20 pts** G is a graph with maximum degree Δ.

   Prove that you can properly edge-colour G with $c\Delta$ colours so that no 4-cycles and no 6-cycles are 2-coloured. Make c as small as you can get it. Of course, it should be smaller than the bound for acyclic edge-colouring that we already saw.

   Hint: Use entropy compression. Intuitively, short cycles are more likely to be 2-coloured than long cycles. So it makes sense to prioritize 4-cycles.

   Comment: It is possible to extend this argument to bound the acyclic edge-chromatic number but that is not required for this assignment.