**CSC 2427** Spring 2004, Assignment #1

**Due:** Feb 24 at the *beginning* of class.

1. A $d$-cycle is a collection of $d$ different cycles that all contain a particular vertex $v$. (Some or all cycles might also intersect on other vertices and edges as well.) For each $d > 1$, determine a probability $q$ such that for every constant $\epsilon > 0$, $G_{n,p}$ a.s. has no $d$-cycle when $p = (1-\epsilon)q$ and $G_{n,p}$ a.s. has a $d$-cycle when $p = (1+\epsilon)q$.

2. A *pendant path* in a graph is a path where every internal vertex has degree exactly two in the graph. (I.e., no internal vertex belongs to a non-path edge.) Prove that for every $t \geq 2$ there exist constants $\epsilon, r > 0$ such that if all pendant paths of length at least $t$ are removed from $G_{n,p}$ where $p = (1+\epsilon)/n$, then a.s. what remains will have no component of size greater than $r \log n$.

   (We only remove pendant paths of the original graph; i.e. we do not iterate the removal procedure.)

3. The chromatic number of a hypergraph is the minimum number of colours neccessary to colour the vertices so that no hyperedge is monochromatic (i.e. within a hyperedge, it is permissable for some vertices to have the same colour so long as not all of them do).

   Consider a random 3-uniform hypergraph on $n$ vertices in which each of the $\binom{n}{3}$ potential hyperedges is present independently with probability 1/2. Determine a function $f(n)$ such that a.s. the chromatic number of that random hypergraph is $f(n)(1+o(1))$.

   ("3-uniform" means that every hyperedge has size exactly 3.)