CSC 2410 Assignment #3, Spring 2009

Due: Monday Apr 6

You may consult the text. You may not consult any other books and materials. You may not consult with each other - you can only consult with the instructor and the TA for this course.

Of course, each problem requires a well-written proof. Proofs that are unnecessarily lengthy might not get full marks, even if they are correct. And they might not be read thoroughly by the grader.

1. **(5 pts)** $\overline{C_7}$ is the complement of the 7-cycle. Either find a planar embedding of $\overline{C_7}$ or find a K_5- or $K_{3,3}$-minor in $\overline{C_7}$.
2. **(10 pts)** 7.1.26 from West.
3. **(10 pts)** Consider a planar graph G formed in the following manner:

   You have a collection of line segments in the plane, such that no 3 lines intersect at the same point. The intersection points of the segments are vertices. Each partial segment running from one intersection point to the next along a line is an edge.

   Prove that G is 3-colourable.
4. **(15 pts)** 6.2.9 from West.
5. **(15 pts)** 7.2.19 from West.
6. (20 pts) In this problem, you may not apply the Four Colour Theorem.

   (a) Prove that if G is a connected planar graph such that (i) G has maximum degree at most 5, and (ii) G has at least one vertex of degree less than 5, then $\chi(G) \leq 4$.

   (b) Prove that if G is a 5-regular planar graph then $\chi(G) \leq 4$.

   In both parts, give an efficient algorithm to 4-colour G.