

CSC 2410, Spring 2009 Assignment #1

Due: Monday Feb 9. Instructions for handing in the assignment will appear on the course web site.

You may consult the text. You may not consult any other materials.

For the first three problems, you may consult with each other but you must each write your own solution. For each problem, list all students with whom you discussed the problem.

For the last two problems, **you may not consult with each other** - you can only consult with the instructor and the TA for this course.

Of course, each problem requires a well-written proof. Proofs that are unnecessarily lengthy might not get full marks, even if they are correct. And they might not be read thoroughly by the grader.

Problem 1: (15 pts) In this problem, you will design an algorithm for the Minimum Spanning Tree problem. First, consider the following algorithm:

We will iteratively grow a forest. Initially, the forest contains no edges, and so each component is a single vertex. In each iteration, for each component C , we choose a least-weight edge from all edges joining C to $G - C$ and add it to the forest. (Possibly, one edge will be chosen by two different components.) So typically many edges are added in each iteration.

(a) Determine a good way to break ties, i.e. to choose when more than one edge is tied for least-weight amongst those edges from C to $G - C$. Then prove that the algorithm will always produce a minimum spanning tree.

(b) Show that this algorithm takes at most $O(m \log n)$ time, where n is the number of vertices and m is the number of edges.

(c) **Fact:** Prim's Algorithm can be implemented to run in $O(m + n \log n)$ time, using Fibonacci heaps.

Using this Fact, combine Prim's Algorithm with the algorithm from parts (a,b) to obtain an minimum spanning tree algorithm that runs in $O(m \log \log n)$ time.

The following problems are all taken from the 2nd edition of West:

(10 pts) 1.1.21

(10 pts) 1.3.34 (see definitions 1.1.35, 1.2.12, 1.3.1 and 1.3.22). Hint: show that if u, v are adjacent then they must have the same degree.

(10 pts) 3.1.37 (see definitions 1.2.17 and 3.1.1)

(15 pts) 3.3.29 (see definition 3.3.11)