CSC 2410, Spring 2012 Assignment #1

Due: Wednesday Feb 8. Either submit by email before midnight, or slip it under my office door (SF3301C) before I arrive Thursday morning.

You may consult the text. You may not consult any other materials.

For the first three problems, you may consult with each other but you must each write your own solution. For each problem, list all students with whom you discussed the problem.

For the last three problems, **you may not consult with each other** - you can only consult with the instructor and the TA for this course.

Of course, each problem requires a well-written proof. Proofs that are unneccessarily lengthy might not get full marks, even if they are correct. And they might not be read thoroughly by the grader.

Problem 1: (20 pts) Suppose that you have a graph G with non-negative weights on its edges, and that you have already found a Minimum Spanning Tree T of G. Someone updates G by adding a new vertex to it, along with non-negative weighted edges from that vertex to some of the original vertices. Give an algorithm to find a Minimum Spanning Tree of the updated graph which is asymptotically faster than just applying Prim's algorithm to the new graph.

Anything asymptotically faster than the worst-case running time of Prim's algorithm will get full marks. You will get some bonus marks if your algorithm runs in $O(n)$ time, where n is the number of vertices.

(Of course, you must prove that your algorithm works.)

If you aren't familiar with the Minimum Spanning Tree problem, read Section 2.3 of West.

The following problems are all taken from the 2nd edition of West:

(15 pts) 1.1.20

(30 pts) 3.3.29 (see definition 3.3.11)

(20 pts) 1.3.34 (see definitions 1.1.35, 1.2.12, 1.3.1 and 1.3.22). Hint: show that if u, v are adjacent then they must have the same degree.

(10 pts) 3.1.28

(20 pts) 3.1.37 (see definitions 1.2.17 and 3.1.1)