The effect of variable elastic topologies on the structure of ocular dominance and orientation maps.

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1 Introduction

Elastic net-type models currently give the best match with experimental data regarding the structure of ocular dominance and orientation columns. However, the standard elastic net model assumes a particular form of cortical interaction, and the effect on the resulting maps of varying this form has not been explored. Here we show that using different forms can have important consequences for map development and structure.

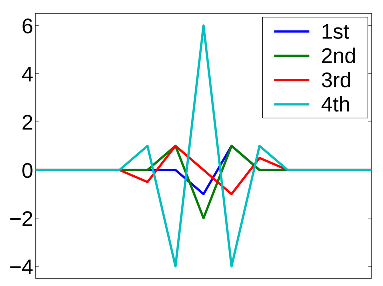
2 The elastic net model

The elastic net minimises an energy function E which trades off coverage and continuity:

$$E\left(\{\mathbf{y}_m\}_{m=1}^{M}, K\right) = \underbrace{-\alpha K \sum_{n=1}^{N} \log \sum_{m=1}^{M} e^{-\frac{1}{2}\left\|\frac{\mathbf{x}_n-\mathbf{y}_m}{K}\right\|^2}}_{\text{coverage term (receptive fields)}} + \underbrace{\frac{\beta}{2}\left\|\mathbf{D}\mathbf{Y}^T\right\|^2}_{\text{continuity term (cortical interaction)}}$$

- $\mathbf{x}$: Feature points
- $\mathbf{y}, \mathbf{Y}$: Cortical receptive field locations
- K: Receptive field width $\begin{cases}\text{annealed: slowly reduced}\\ \text{nonannealed: kept constant}\end{cases}$
- α, β: Weighting of coverage and continuity terms
- $\mathbf{D}$: Generalised definition of continuity (or cortical interaction):

Type	Stencil	Effec. cortical interaction
1st order	$(0,-1,1)$	All-excitatory
2nd order	$(1,-2,1)$	Mexican hat
3rd order	$\frac{1}{2}(-1,2,0,-2,1)$	
4th order	$(1,-4,6,-4,1)$	



3 Fourier analysis of cortical interaction term

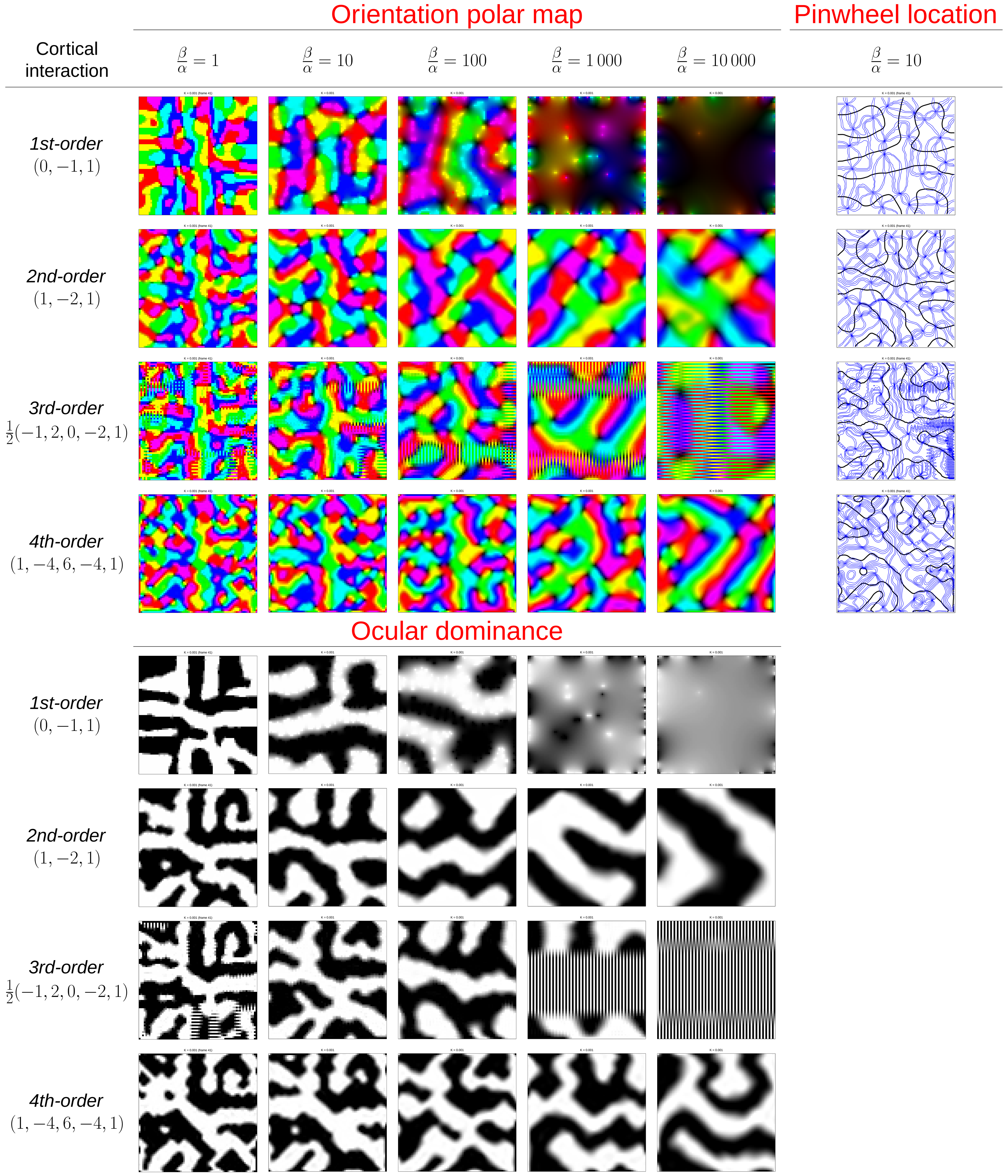
For a continuous 1D net, the interaction term is a filtering or convolution. In the Fourier domain (x: cortical location, z: frequency):

$$\text{cortical interaction term} = \int |\mathbf{D}y|^2\,dx = \int |\mathcal{F}(\mathbf{D}y)|^2\,dz = \int |\mathcal{F}(\mathbf{D})\mathcal{F}(y)|^2\,dz = \int \underbrace{|\mathcal{F}(\mathbf{D})|^2}_{\text{filter}}\,\underbrace{|\mathcal{F}(y)|^2}_{\text{original spectrum}}\,dz.$$

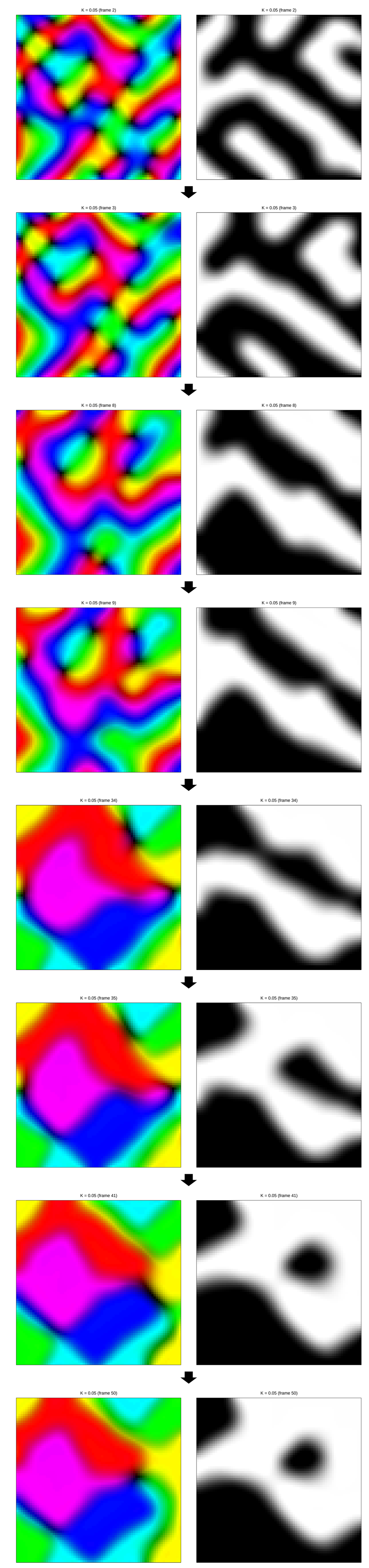
Since the types of interaction $\mathbf{D}$ above are band-pass filters, minimising the interaction term results in suppressing the frequencies in that band. Besides, the higher the interaction order, the higher the maximum frequency allowed.

4 2D net: ocularity + orientation + 2D retinotopy

Effect of the continuity term on map structure (with annealing)



Pinwheel annihilation (without annealing, *2nd-order* interaction)



Summary of results

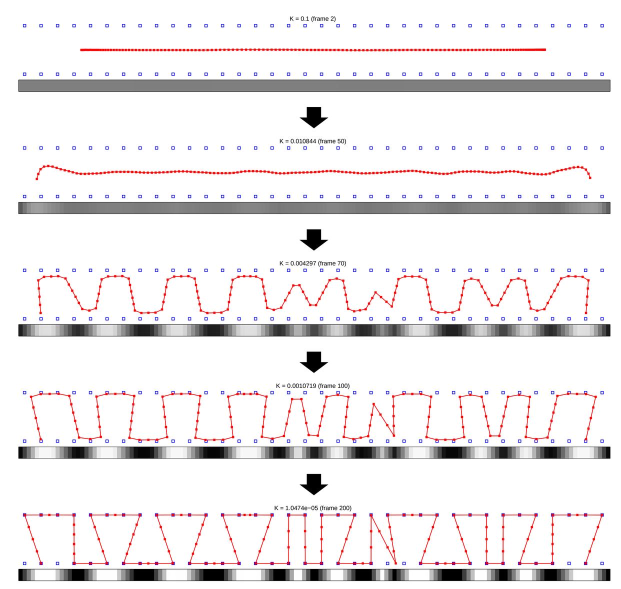
- Higher-order cortical interactions give narrower columns.
- $\frac{\beta}{\alpha}$: $\begin{cases}\text{very low: all interaction types behave similarly}\\ \text{high: wider columns}\\ \text{very high: prevents segregation for } \textit{1st-order}.\end{cases}$

  Previous studies considered only the *1st-order* interaction with very low $\frac{\beta}{\alpha}$.
- Pinwheel location: $\begin{cases}\textit{1st-order}\text{: mostly away from ocular dominance borders}\\ \text{order} > 1\text{: many on ocular dominance borders.}\end{cases}$
- With annealing, initial maps arise with a specific stripe width that remains fixed.
- Without annealing, the initial stripes widen and there is pinwheel annihilation.
- The *3rd-order* interaction can show discretisation artifacts.

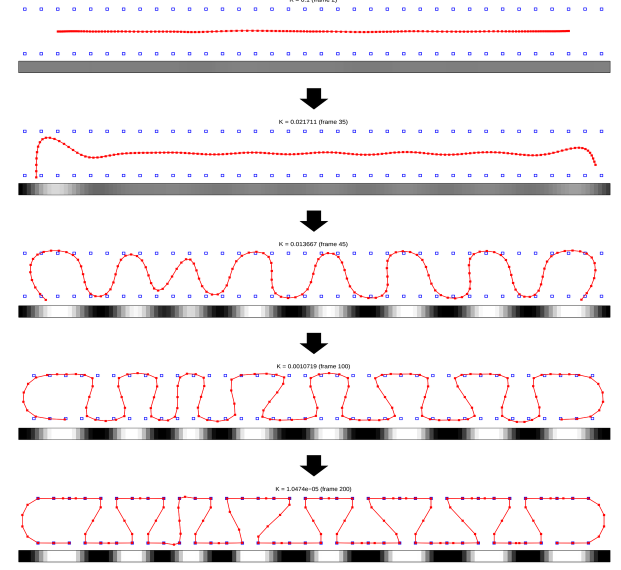
5 1D net: ocularity + 1D retinotopy

With annealing

1st-order $(0,-1,1)$

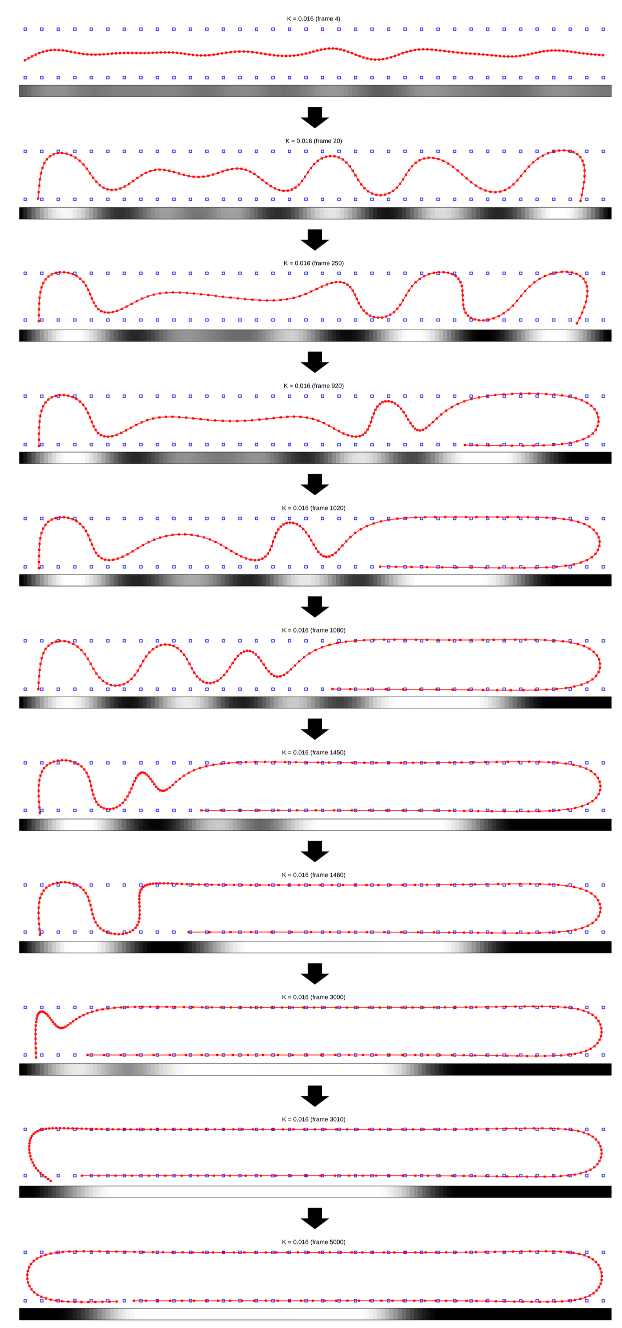


2nd-order $(1,-2,1)$



Without annealing

2nd-order $(1,-2,1)$



- With annealing, the development sequence is:
  1. Net unselective to ocularity.
  2. Initial wave appears (bifurcation of energy function).
  3. Final state is a set of stripes.

  The *2nd-order* interaction enforces smoother transitions between ocularity values (observe rounded corners) than the *1st-order* one (observe sharp corners).
- Without annealing, there exists loop elimination for a range of values of K for cortical interactions of order > 1 (cf. pinwheel annihilation).
- For very small $\frac{\beta}{\alpha}$ all interaction types give a similar map structure.

6 Conclusions

- More general forms of cortical interaction can be introduced into the elastic net.
- Map structure and development is sensitive to the type of cortical interaction.
- Quantitative analysis of these differences is in progress.
- This provides additional flexibility for the algorithm to account for differences between species, and between different regions of V1.