Due: By 6:00pm on Thursday 16 November, in the CSC 373H drop box in room BA 2220. **Worth**: 7.5%

1. [5 marks]
   Consider the Shortest Paths problem of Section 6.8 of the text. We have a directed graph $G = (V, E)$, and for each edge (v, w) we have a cost (or weight, or length) $c_{vw} \in \mathbb{R}$. Some costs may be negative. There are n vertices, and we are given a particular vertex t. We want to find, for every vertex v, the cost of a shortest (that is, least-cost) path from v to t. The text assumes that there are no negative-weight cycles.

   Let us remove this assumption. In this case, if there is a part from v to t that contains a negative weight cycle, we can get arbitrarily small weight paths from v to t by repeating that cycle as often as we want. So we change the problem to say that we want to find the cost of a smallest weight path from v to t, amongst only those paths that do *not* contain a negative-weight cycle. For convenience for this question, we will assume that all n^2 edges are present.

   One possible approach to this problem is to define the array OPT as follows:
   For all $0 \leq i \leq n$, for all vertices v,
   $\text{OPT}(i, v) =$ the least cost of a path among all those paths from v to t that use at most i edges and that do not contain a negative weight cycle.

   It is reasonable to conjecture that assertion (6.23) from the text is still true in our setting.
   Conjecture: For every $0 < i \leq n$, for every vertex v,
   $\text{OPT}(i, v) = \min\{\text{OPT}(i-1, v), \min_{w \in V}\{\text{OPT}(i-1, w) + c_{vw}\}\}$.

   Prove or disprove this conjecture. (That is, either prove that the Conjecture holds for every weighted graph, or prove that it is not true that the Conjecture holds for every weighted graph.)

2. [15 marks] Do Problem 8 in Chapter 6 of the book (destroying robots).

3. [15 marks] Do Problem 19 in Chapter 6 of the book (untangling signal).

4. [5 marks] Do Problem 4 in Chapter 7 of the book (prove/disprove question).

5. [5 marks] Do Problem 5 in Chapter 7 of the book (prove/disprove question).