BILS:
MATLAB package for solving Box-Constrained Integer Least Squares Problems

User's Guide

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1 Introduction

Let the sets of all real $m \times n$ matrices be denoted by $\mathbb{A}^{m\times n}$ and the sets of real and integer n-vectors by $\mathbb{R}^n$ and $\mathbb{Z}^n$, respectively. Given y $\in \mathbb{R}^m$ and A $\in \mathbb{R}^{m\times n}$ with full column rank, one wants to solve the minimization problem

$$\min_{x\in\mathbb{Z}^n} \|y - Ax\|_2^2. \tag{1}$$

We refer to (1) as the *integer least squares problem*. In some applications, the integer vector x is constrained to a box:

$$B = \{x \in \mathbb{Z}^n : l \leq x \leq u, l \in \mathbb{Z}^n, u \in \mathbb{Z}^n\}. \tag{2}$$

Then, one wants to solve

$$\min_{x\in\mathcal{B}} \|y - Ax\|_2^2. \tag{3}$$

We refer to (3) as the *box-constrained integer least squares (BILS) problem*. This MATLAB package provides the solution to the BILS problem. The purpose of this document is to show how to use this package. For the theory and algorithms implemented in this package, see the paper "Solving Box-Constrained Integer Least Squares Problems".

2 System Requirements

This package has been fully tested on Windows XP and Linux with MATLAB 7.x, and should work on any platform supporting MATLAB. For the system requirements of running MATLAB, refer to `http://www.mathworks.com`.

3 Installing Package

The package can be downloaded from `www.cs.mcgill.ca/~chang/software.php`. The package is provided as a compressed file with extension "zip" and "tar.gz". To extract the package, an uncompress tool should be used, such as "winzip" on Windows or "gzip" on Linux. To use the functions of the package, enter the directory where the package was extracted or add that directory to the MATLAB path.

4 Commercial Use and Citation

BILS is a free software package available on the Internet. It can be included in commercial packages. We ask users to give proper credit to the authors by citing this document or the document which describes the theory and algorithms. This package is copyrighted but not trademarked. If modifications made to a routine affects the interface, functionality or accuracy of the package, then the name

of the routine should be changed. Any modification to this package should be noted in the modifier's documentation.

5 Support

We will gladly answer questions regarding this package. Reports of errors or poor performance will gain immediate attention from the developers. Any comments or suggestions for improving the code or the documentation is also welcome. It may still be possible to improve the efficiency of the package by using programming tricks or MATLAB built-in functions, but for research purposes, we try to keep to code simple and clean. Errors, comments, and descriptions of interesting applications should be sent to:


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6 Routines

In this section, we will introduce the inputs and outputs of some routines of this package. Each routine can be used separately without invoking the main function `BILS.m`. Those only interested in an example on how to use `BILS.m` can skip this section and start directly by reading the example file `bilsExample.m`.

6.1 reduction_bils.m

Reduces the general BILS problem to an upper triangular one. The permutations of the columns of the upper triangular are determined such as to reduce the search range for the optimal solution to the BILS problem. The MATLAB function is:

```
function [R P yt l u zh] = reduction_bils(A,y,l,u)
```

Input arguments

A — m by n real matrix with full column rank
y — m-dimensional real vector
l — n-dimensional lower-bound integer vector
u — n-dimensional upper-bound integer vector

Output arguments

R	—	n by n real upper triangular matrix
P	—	n-dimensional integer permutation vector
yt	—	n-dimensional real vector. It's the updated value of y
l	—	n-dimensional updated lower-bound integer vector
u	—	n-dimensional updated upper-bound integer vector
zh	—	n-dimensional integer vector. It's the Babai integer point

6.2 search_bils.m

Produces the solution to the upper triangular BILS problem $\min_{\substack{z\in\mathbb{Z}^n \\ l\leq z\leq u}} ||y-Rz||_2^2$ by a search algorithm. The MATLAB function is:

```
function[zh found]= search_bils(R,y,l,u,B)
```

Input arguments

R	—	n by n real nonsingular upper triangular matrix
y	—	n-dimensional real vector
l	—	n-dimensional lower-bound integer vector
u	—	n-dimensional upper-bound integer vector
B	—	positive real number. It's the ellipsoid bound

Output arguments

zh	—	n-dimensional integer vector. It's the solution to the upper triangular BILS problem.
found	—	flag. A value of 1 indicates that the search found a point. Other values indicates that no point was found.

6.3 BILS.m

Produces the solution to the general BILS problem (3). This is the main function of the package. The MATLAB function is:

```
function z = BILS(A,y,l,u)
```

Input arguments

A	—	m by n real matrix with full column rank
y	—	m-dimensional real vector
l	—	n-dimensional lower-bound integer vector
u	—	n-dimensional upper-bound integer vector

Output argument

z	—	n-dimensional integer vector. It's the solution to the BILS problem.

6.4 Examples

We provide a script M-file, `bilsExample.m`, as a simple example of using this package to solve a BILS problem.

7 Troubleshooting

Some of the problems that can be encountered by a user are listed below.

7.1 Common Errors

A user should always carefully read the leading comments of a routine before using it. The leading comments describes the functionality of the routine and gives a description of it's input and output arguments. For the benefit of users, we list common programming errors in calling a routine. These errors may cause the BILS routines or MATLAB to report a failure, or may lead to wrong results without a warning message.

- Wrong number of arguments
- Arguments in wrong order
- Wrong dimensions for an array argument
- The input matrix is rank deficient
- MATLAB path is not set up correctly

8 Poor Performance in efficiency

The BILS problem is NP-hard. Therefore, if the dimension of a BILS problem is large, the computation can be very time-consuming. We also note that MATLAB is slower than some high level programming languages, such as C and C++.

9 Integer overflow

If an integer number produced in the computation is outside of the interval $[-2^{53}+1, 2^{53}-1]$, then it's floating point representation may not be accurate. This may lead to wrong integer solutions. However, for many practical applications, integer overflow is not a concern. Version 1.0 of this package does not check for integer overflow and does not generate a warning message if it occurs.