CSC 363 - Summer 2005
Assignment 3

due on Tuesday, July 19th, at 6pm

Problem 1 [5p]

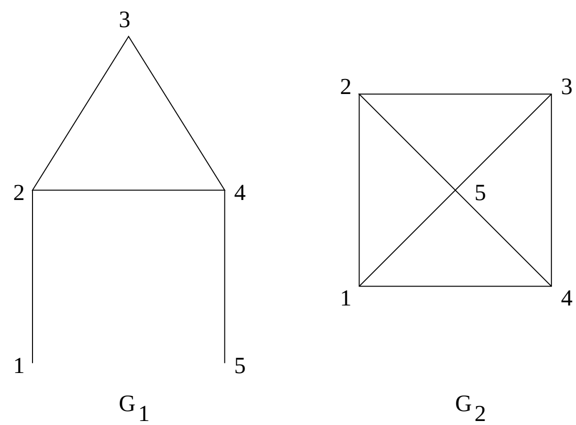
This is part of problem 7.25 in Sipser. Let

$$U = \{\langle N, w, 1^t\rangle \mid N \text{ is a NTM that accepts } w \text{ within } t \text{ steps}\}.$$

Show that U is **NP-hard**.

Problem 2 [35p]

In this problem, we concentrate on vertex covers. Let $G = (V, E)$ be a graph on n vertices. A *vertex cover* of G is a set of vertices $S \subseteq V$ such that every edge has at least one endpoint in S. Note that V itself is a trivial vertex cover of size n. In general, it is difficult to find a vertex cover of *minimum* size. For example, $\{2, 4\}$ is a minimum size vertex cover for G_1 and $\{1, 3, 5\}$ is a minimum size vertex cover for G_2. We know that $VC = \{\langle G, k\rangle \mid G \text{ has a vertex cover of size } k\}$ is **NP-complete**.



(a) [5p] Prove that $L_1 = \{\langle G\rangle \mid \text{the smallest vertex cover of } G \text{ has size } n - 5\}$ is in **P**.

(b) [10p] We say that a vertex cover S is *minimal* if removing any vertex of S results in a set which is no longer a vertex cover. For example, $\{1, 3, 4\}$ is a minimal vertex cover of G_1, and $\{1, 2, 3, 4\}$ is a minimal vertex cover of G_2. Note that a minimal vertex cover is not necessarily a vertex cover of minimum size.

Let $L_2 = \{\langle G, F\rangle \mid F \text{ is the set of all minimal vertex covers of } G\}$. For example, if $F = \{\{1, 2, 3, 4\}, \{1, 3, 5\}, \{2, 4, 5\}\}$ and $F' = \{\{1, 3, 5\}, \{2, 4, 5\}\}$, then $\langle G_2, F\rangle \in L_2$ but $\langle G_2, F'\rangle \notin L_2$. Similarly, if $F = \{\{1, 3, 4\}, \{2, 3, 5\}, \{2, 4\}\}$, then $\langle G_1, F\rangle \in L_2$.

Prove that L_2 is in **coNP**.

(c) [10p] Assume that we had a polynomial time algorithm A for deciding VC. That is, $A(\langle G, k\rangle) = 1$ iff G has a vertex cover of size k. Give a polytime algorithm B that uses A, and that, on input $\langle G\rangle$, outputs a set $S \subseteq V$ which is a minimum size vertex cover of G.

Argue that your algorithm B works as it should. If applicable, provide a loop invariant which is easy to verify. Analyze the running time of B.

(d) [10p] Prove that $L_3 = \{\langle G\rangle \mid G \text{ has a vertex cover of size } n/2 \text{ or less}\}$ is **NP-complete**.

Note. For the purposes of this question, you may only assume that VC is **NP-complete**. You may **not** use any other language, even if it was discussed in class or tutorial. In particular, you may not use any languages talking about cliques and/or independent sets.