CSC 363 - Summer 2005
Assignment 2

due on Tuesday, June 28th, at 6pm

Problem 1 [5p] For every positive integer t, we say that a language L is *decidable with lag* t if there exists a TM M deciding L, which also satifies the requirement that on every input w, M halts in at most $|w| + t$ steps. For example, the arguments presented in the solution to question 4c from Assignment 1 show that every finite language is decidable with lag 1.

Consider a table whose rows are labelled with TMs, whose columns are labelled with TM encodings, and where the entry at $(M_i, \langle M_j \rangle)$ contains a 1 if M_i accepts $\langle M_j \rangle$ in at most $|\langle M_j \rangle| + t$ steps, and a 0 otherwise.

Use a diagonalization argument to show that for every t, there exists a language L which is decidable in general, but not decidable with lag t.

Problem 2 [10p] Let $M_1, M_2, \ldots, M_i, \ldots$ be the list of all TMs in lexicographical order.

For every positive integer k, define $f(k)$ to be the index in the above list of the k-th TM M such that $L(M) = \emptyset$. Formally,

$$f(k) = \max\{i \ : \ \exists J \subseteq \{1 \ldots i-1\} \text{ such that } (|J| = k-1 \text{ and } \forall j \in \{1 \ldots i-1\}, L(M_j) = \emptyset \Leftrightarrow j \in J)\}$$

Notice that f is well defined for every k, since there are infinitely many TMs M with $L(M) = \emptyset$. For example, if $L(M_2) = L(M_5) = \emptyset$ and $L(M_1), L(M_3), L(M_4)$ are not empty, then $f(1) = 2$ and $f(2) = 5$.

Prove that f is not computable.

Problem 3 [10p] This is problem 5.14 in Sipser. Let L be the language of all encodings $\langle M, w \rangle$ such that the TM M on input w attempts to move its head left of the initial position (i.e. left of the position of the leftmost symbol in the input string). Show that L is undecidable.

Note: TMs in Sipser have one-way infinite tapes, while our TMs have a two-way infinite tapes. The statement above is relevant for our model.

Problem 4 [10p] This is problem 5.10 in Sipser first edition, and 5.24 in Sipser second edition. Let

$$J = \{0x : x \in A_{TM}\} \cup \{1y : y \in \overline{A_{TM}}\}.$$

Show that neither J nor $\overline{J}$ is recognizable.