# CSC363 - Summer 2005
# Assignment 1

due on Tuesday, June 7th, at 6pm

**Problem 1** [10p]

Give a regular Turing Machine deciding the language $L = \{ww \mid w \in \{0,1\}^*\}$. Begin with a high-level description of how the machine works, followed by a formal description. To specify the transition function, you may use a table or a diagram. Also include informal descriptions of the states of your machine.

**Problem 2** [10p]

A 2-head Turing Machine has a single two-way infinite tape, and 2 heads which can operate on the tape independently of each other. The transition function of a 2-head TM is of the form

$$\delta(q, c_1, c_2) = (q', c_1', D_1, c_2', D_2)$$

where $q, q'$ are states, $c_1, c_2, c_1', c_2'$ are characters of the tape alphabet, and $D_1, D_2$ are directions Left or Right. This transition means that, when the TM is in state $q$, head number 1 is reading $c_1$ and head number 2 is reading $c_2$, the machine moves to state $q'$, head number 1 writes $c_1'$ and moves in the direction $D_1$, and head number 2 writes $c_2'$ and moves in the direction $D_2$.

We adopt the following convention: if the two heads are reading the same cell prior to a transition, and they attempt to write distinct characters to the same cell during that transition, the character written by head number 1 will end up on the tape.

Show that 2-head TMs recognize the same class of languages as regular TMs. In your proof, you may give implementation-level descriptions of TMs (descriptions in English of how the machine moves its head and how it stores data on its tape).

**Problem 3** [10p]

This is problem 4.18 from p.170 in Sipser. Let $A$ and $B$ be two disjoint co-Turing-recognizable languages (meaning that their complements are Turing-recognizable, see p.167). Show that there exists a decidable language $C$, such that $A \subseteq C$ and $B \subseteq \overline{C}$.

**Problem 4** [15p]

Let $A_1, A_2, \ldots, A_i, \ldots$ be a list of some languages such that for every $i$, $A_i$ is decidable (i.e. for every $i$, there exists a decider $M_i$ such that $A_i = L(M_i)$). Let

$$A = \bigcup_{i \geq 1} A_i$$

**(a)** [5p] Prove that $A$ is recognizable.

**(b)** [5p] Can $A$ be recognizable but not decidable? Prove your answer.

**(c)** [5p] Let $k$ be some fixed integer. Consider the language

$$L_k = \{w \in \{0,1,\ldots,9\}^* \mid |w| \leq k \text{ and } w \text{ appears in the decimal expansion of } \pi\}$$

In plain English, $L_k$ contains those strings $w$ of length at most $k$ such that $w$ appears in the decimal expansion of $\pi$. We know $\pi = 3.14159265..$, so, for example, $41 \in L_2$ and $926 \in L_4$. Is $L_k$ recognizable? Is $L_k$ decidable?

**(d)** [Bonus 2p] Do you think that the following language is decidable? Explain.

$$L = \{w \in \{0,1,\ldots,9\}^* \mid w \text{ appears in the decimal expansion of } \pi\}$$