CSC363 - Summer 2007
Assignment 1

due on Tuesday, June 5th.

Problem 1 [20p]

Let L be the language of strings w over the alphabet $\{0, 1\}$ such that w contains exactly twice as many 0s as 1s or exactly twice as many 1s as 0s. For instance, $\epsilon, 010, 110101 \in L$ but $0, 10 \notin L$.

Give an implementation level and a formal level description of a decider TM for L.

Problem 2 [20p]

So far, we studied Turing Machines that only accept or reject strings. For this problem, we will consider TMs that output something other than a simple yes/no answer. Specifically, we want to compute addition.

Given a string $w \in \{0, 1\}^*$, let $\text{int}(w)$ denote the non-negative integer that has w as its binary representation. We allow leading 0s in w, so, $\text{int}(00110) = \text{int}(110) = 6$, $\text{int}(10001) = 17$. By convention, we define $\text{int}(\epsilon) = 0$ (where ϵ is the empty string).

Let f be the addition function, defined over the alphabet $\{0, 1, \#\}$ as follows. If $w = a\#b$ for $a, b \in \{0, 1\}^*$, then $f(w)$ is equal to the binary representation of $\text{int}(a) + \text{int}(b)$ with no leading 0's. If w is not of the form above (i.e. w contains either 0 or 2 or more $\#$), then define $f(w) = \epsilon$, the empty string. So, for example, $f(00011\#) = 11$, $f(100\#1110) = 10010$, $f(000\#1) = 1$, $f(0\#) = \epsilon$ and $f(\epsilon) = f(0\#0\#01) = \epsilon$.

Give a Turing Machine that computes f as follows. On input w, the machine ends up in q_{accept} with nothing but $f(w)$ on the tape and the head on the first symbol of $f(w)$.

First, describe your TM in words, breaking it down in stages. For example, a stage could be "check that the input contains exactly one #". Then, give implementation-level details about how the TM performs each stage. You should not care about efficiency (e.g. how many times the TM is scanning the tape, or how many tape symbols you're using) as much as about clarity.

Problem 3 [20p]

A 2-head Turing Machine has a regular one-way infinite tape and 2 heads that can operate independently of one another. The transition function of a 2-head TM is of the form

$$\delta(q, a, b) = (q', a', b', D_1, D_2)$$

where q, q' are states, a, b, a', b' are tape characters, and D_1, D_2 are directions Left or Right. This transition says that, if the TM is in state q, head 1 reads a and head 2 reads b, the next state is q', head 1 replaces a by a' and moves in the direction D_1, and head 2 replaces b by b' and moves in the direction D_2. We adopt the convention that if the two heads try to simultaneously write something on a tape cell, the character written by head 1 ends up on the tape. As usual, neither head can move off the left end of the tape.

Show that 2-head TMs are equivalent in power to regular TMs. In your proof, give implementation-level descriptions of TMs.

Problem 4 [20p]

Let A and B be two languages over the same alphabet Σ such that $A = \{x \in \Sigma^* : \exists y \text{ such that } xy \in B\}$. For instance, the following languages satisfy this condition: $A = \{\epsilon, 0, 1, 01, 11\}$ and $B = \{01, 11\}$.

Show that if B is decidable, then A is recognizable.

In your proof, give high-level descriptions of TMs.