LCA

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Modeling the 802.11 protocol under different capture and sensing capabilities.

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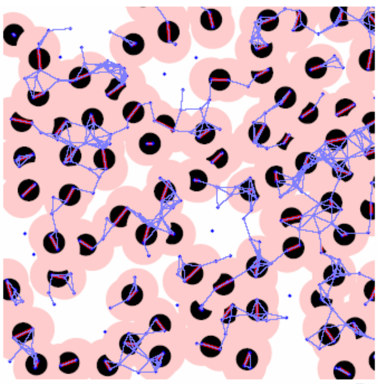
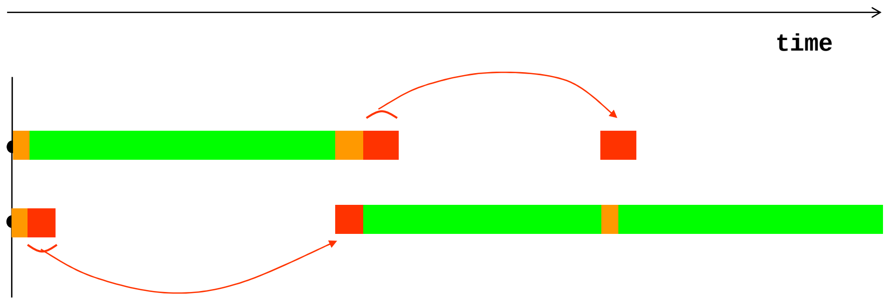
Reaching the limit by MAC algorithms

- Self-organizing Medium Access Control (MAC) algorithms for large wireless multi-hop networks.
- Candidates: **decentralized** MAC protocols using only back-off mechanisms + carrier sensing (à la IEEE 802.11).
- Each transmission takes a footprint free of interferers
- Random packing : Pack as many transmissions concurrently, while using only local information.



Wireless multihop nets: Back-off MACs

- Take continuous back-off distribution (idealized CSMA: no collision)
- Average exchange time $1/\mu$.
- Average backoff duration $1/\lambda$.



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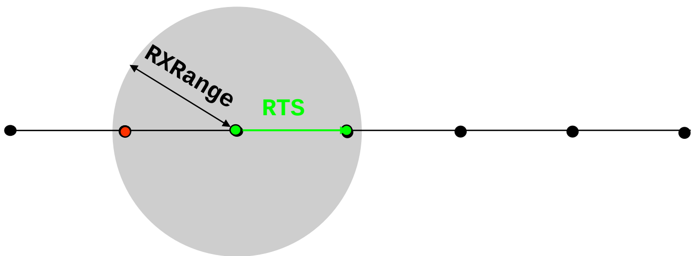
Exclusion Domain

- RTS (Request-To-Send)
- CTS (Clear-To-Send)
- DATA packet
- ACKnowledgment



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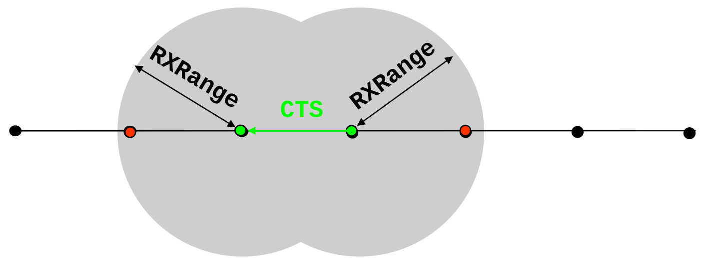

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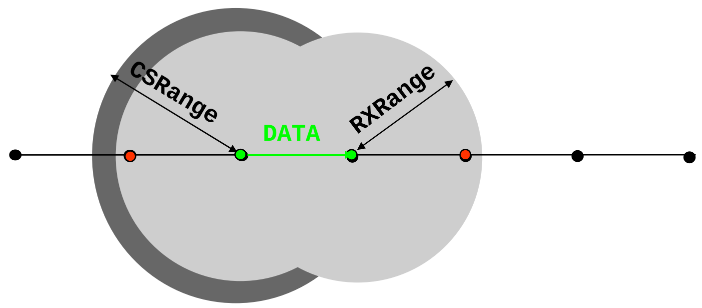
Symmetric Exclusion Domain

- ☐ RTS (Request-To-Send)
- ☐ CTS (Clear-To-Send)
- ☐ DATA packet
- ☐ ACKnowledgment

CSRange ≈ RXRange



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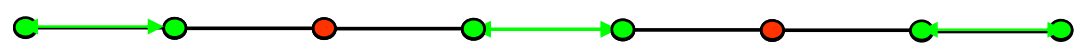
MAC = a packing problem

- Saturated traffic condtions
- Transmission schedule (active and idle links)

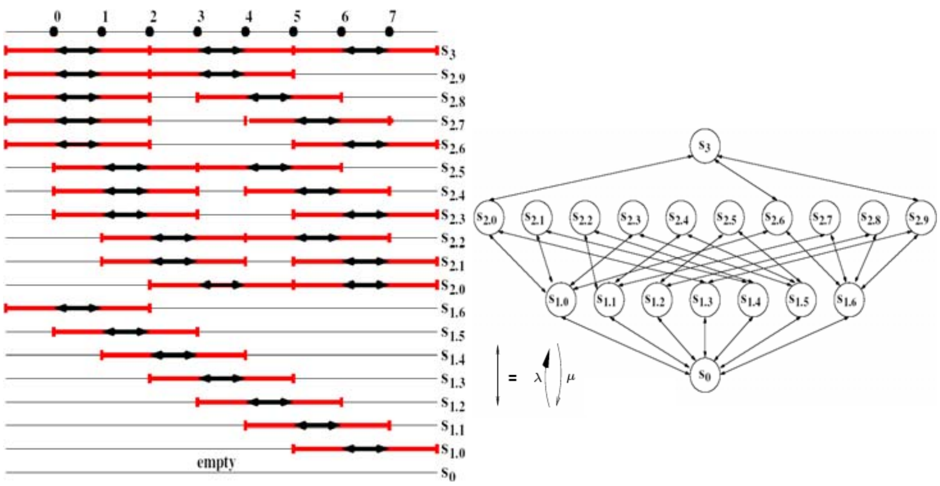


- MAC as a packing problem: To each active link, we associate an interval of length l (Here l = 3) on the line.

- Intervals arrive according to Poisson process of rate λ, depart after having been active during i.i.d. expo(μ) distributed times.
- Reversible Markov chain (= Kelly network) -> product solution

MAC: a packing problem

- Markov chain for any topology

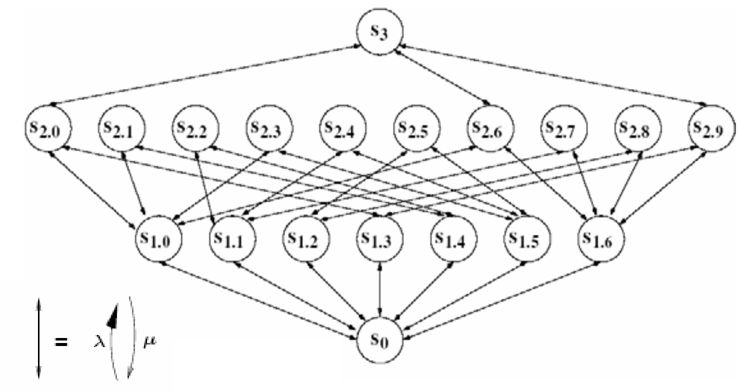


Reversible Markov chain

- ☐ Stationary probability that i links are active

  $\pi(i)$ = Prob (pattern s_{ij} with i active links) = $(\lambda/\mu)^i / \sum_k N(k) (\lambda/\mu)^k$

  with N(k) = number of patterns with k active links (Gibbs measure)
- ☐ When $\lambda > \mu$, Prob(i active links) increases with i.
- ☐ When $\lambda/\mu \to \infty$, Prob(max nr active links) $\to$ 1.



Spatial reuse

- ☐ Spatial reuse with a total number of links L

  Average proportion of active links $\sigma = (\sum_i i\, N(i)\, \pi(i)) / L$.
- ☐ Lemma: Let $m \geq 2$ and $n \geq 0$ be two integers, and $r > 0$ be a real. Let $\{X(k), k \geq 1\}$ be a sequence of discrete random variables with

$$P(X(k)=i) = \binom{km-(m-1)i+n}{i} r^i Z^{-1}$$

  for $0 \leq i \leq k$ and P(X(k) = 0) otherwise, where Z = Z(m,n,r) is a normalizing constant. Then as $k \to \infty$

$$\frac{E[X(k)]}{k} \to \frac{m r y^{m-1}}{1+m r y^{m-1}}$$

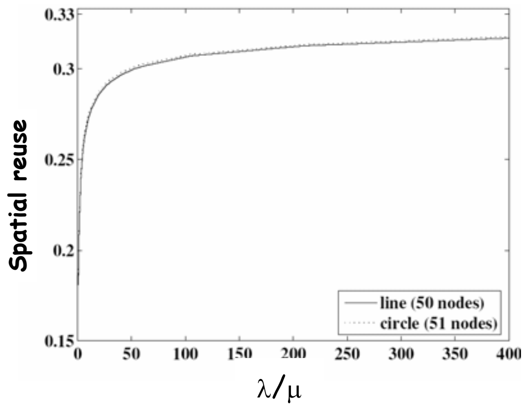
  where y is the positive real root of $1 - y - r y^m = 0$.
- ☐ Here
  - X(k) = level of the Markov chain, with $k = \lfloor (L + l - 1)/l \rfloor$
  - $P(X(k) = i)$ = Prob(level i of the chain) = Prob(i active links) = $N(i)\pi(i)$
  - $r = 2\lambda/\mu$
  - $m = l$
  - $n = (L + l - 1) \bmod l$.
  - for k (and thus L) large, $E[X(k)]/k = (\sum_i i\, N(i)\, \pi(i)) / (L/l) = l\, \sigma$

Spatial reuse

- Spatial reuse with a total number of links L
  Average proportion of active links $\sigma = (\sum_i i\, N(i)\, \pi(i)) / L$
- Theorem: On the infinite line network ($L \to \infty$)

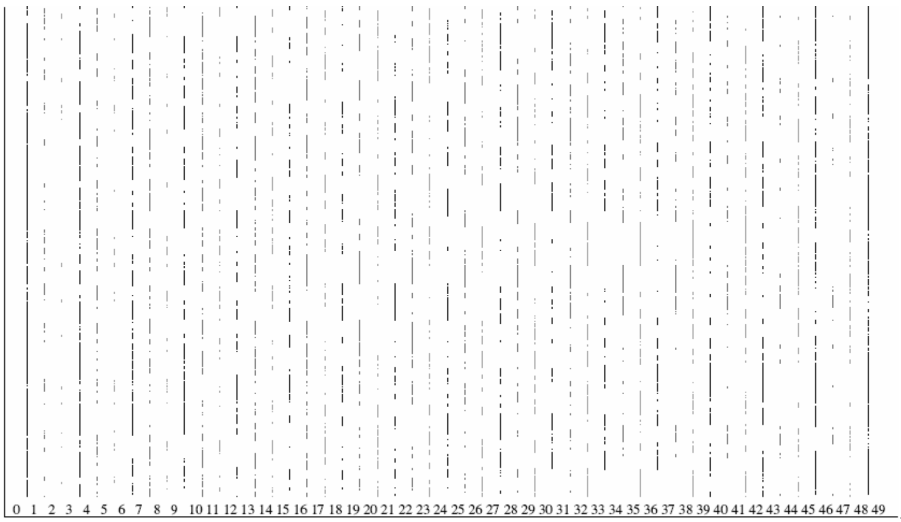
$$\sigma = (l + \mu/(2\lambda y^{l-1}))^{-1}$$

where y is the positive real root of $1 - y - 2y^l \lambda/\mu = 0$.



Wireless multihop nets: Back-off MACs

- Different values of back-off lead to very different activity patterns



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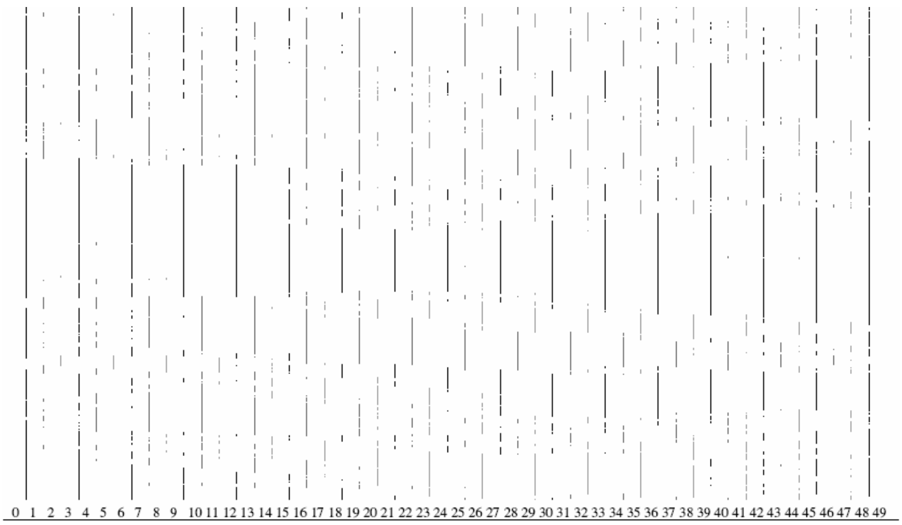
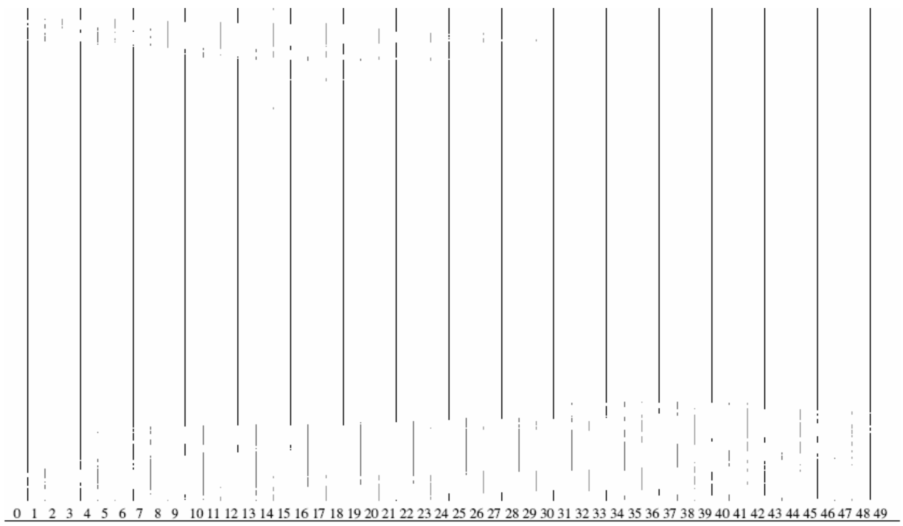


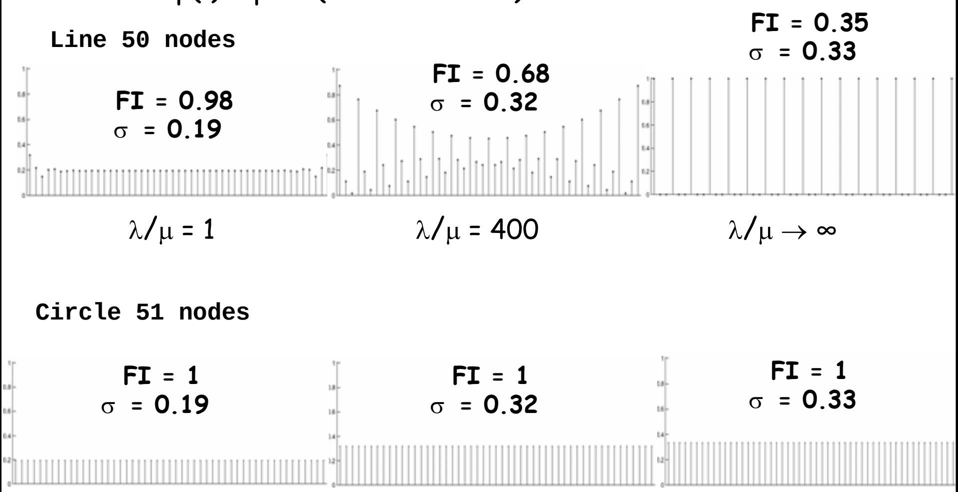
Wireless multihop nets: Back-off MACs

- Different values of back-off lead to very different activity patterns



Fairness vs spatial reuse

- Long term fairness index (Jain): $FI = (\sum_i p(i))^2 / (L \sum_i p^2(i))$ where p(i) = prob(link i is active)



Asymmetric Exclusion Domain

- RTS (Request-To-Send)
- CTS (Clear-To-Send)
- DATA packet
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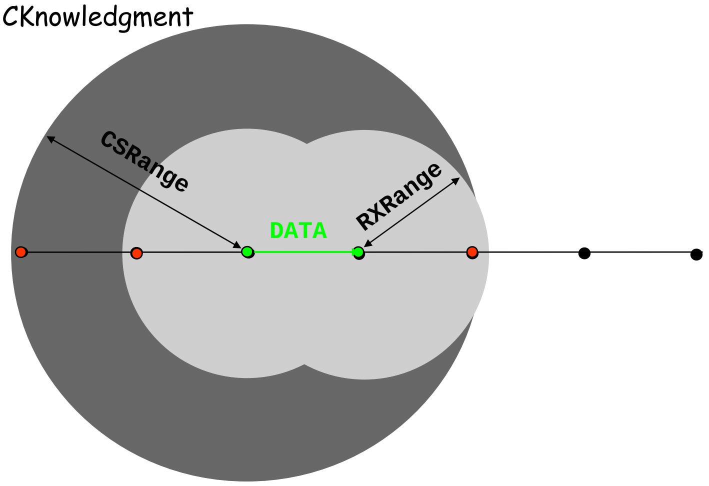
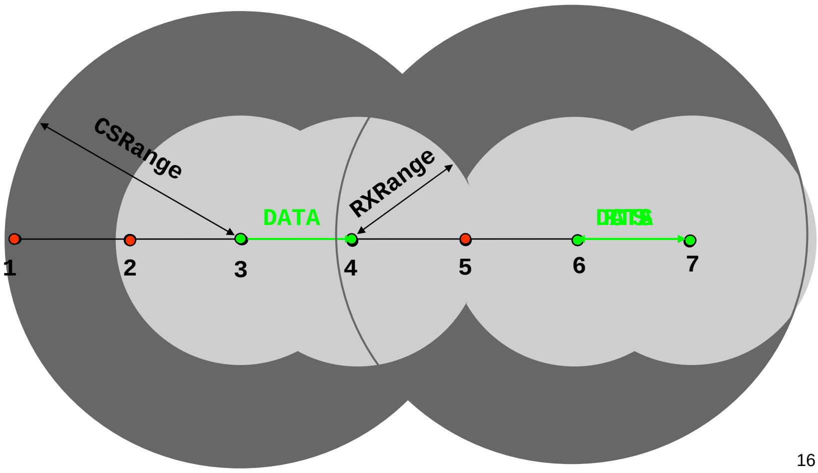
CSRange > RXRange



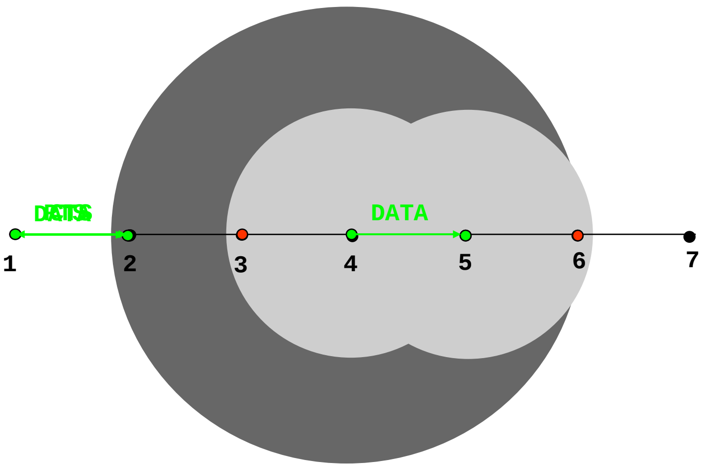
Capture effect

- ☐ The strongest signal arrives first at Node 4.
- ☐ It is captured by Node 4 because P3/P4 > threshold.
- ☐ Node 4 can continue decoding Data from Node 3.



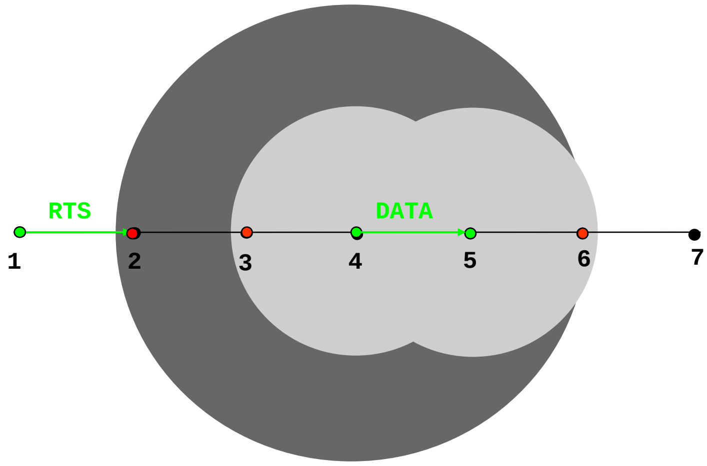
Two capture models: 1) Full Capture

- ☐ The strongest signal arrives second at Node 2.
- ☐ Node 2 can resynchronize on the strongest signal.
- ☐ Node 2 can decode Data from Node 1.



Two capture models: 2) Limited Capture

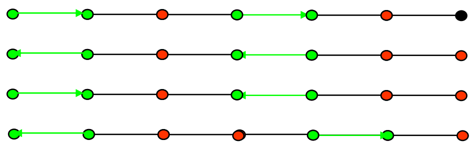
- The strongest signal arrives second at Node 2.
- Node 2 cannot resynchronize on the strongest signal.
- Node 2 cannot decode Data from Node 1.



Full Capture

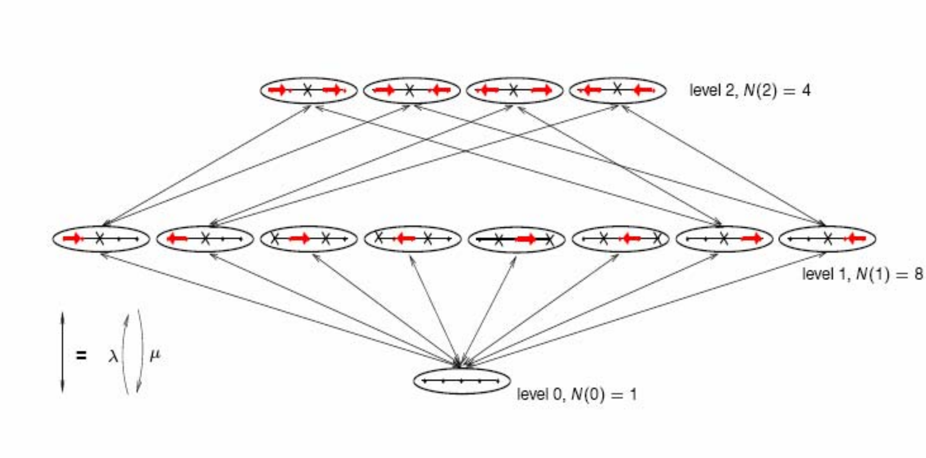
- Symmetric Exclusion Domain: Carrier Sense Range = Receiver Range
  - Can consider non directed links.

- Asymmetric Exclusion Domain: Carrier Sense Range > Receiver Range and Full Capture
  - Must take directed links
  - New connection accepted independently of order of arrival of neighboring connections.



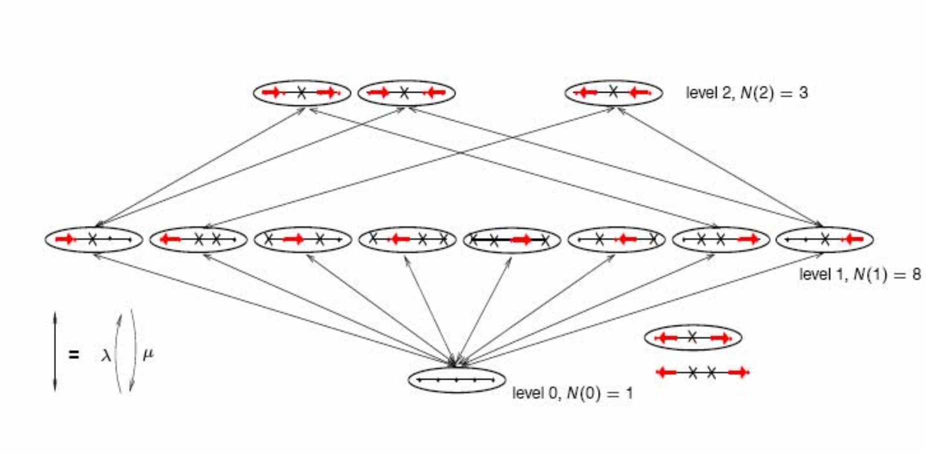
Symmetric Exclusion Domain

- ☐ Reversible Markov Chain



Asymmetric Excl. Domain with Full Capture

- ☐ Reversible Markov Chain



Spatial reuse (symmetric)

- Spatial reuse with a total number of links L
  Average proportion of active links $\sigma = (\sum_i i\, N(i)\, \pi(i)) / L$.
- Lemma: Let m ≥ 2 and n ≥ 0 be two integers, and r > 0 be a real. Let {X(k), k ≥ 1} be a sequence of discrete random variables with

$$P(X(k)=i)=\binom{km-(m-1)i+n}{i} r^i Z^{-1}$$

  for 0 ≤ i ≤ k and P(X(k) = 0) otherwise, where Z = Z(m,n,r) is a normalizing constant. Then as $k \to \infty$

$$\frac{E[X(k)]}{k} \to \frac{m r y^{m-1}}{1+m r y^{m-1}}$$

  where y is the positive real root of $1 - y - r y^m = 0$.
- Here
  - X(k) = level of the Markov chain, with $k = \lfloor (L + l - 1)/l \rfloor$
  - P(X(k) = i) = Proba (level i of the chain) = Prob(i active links) = $N(i)\pi(i)$
  - $r = 2\lambda/\mu$
  - m = l
  - n = (L + l - 1) mod l.
  - $E[X(k)]/k = (\sum_i i\, N(i)\, \pi(i)) / (L/l)$ for k and thus L large = $l\,\sigma$

Spatial reuse (asymmetric, full capture)

- Spatial reuse with a total number of links L
  Average proportion of active links $\sigma = (\sum_i i\, N(i)\, \pi(i)) / L$.
- Lemma: Let m ≥ 2 and n ≥ 0 be two integers, and r > 0 be a real. Let {X(k), k ≥ 1} be a sequence of discrete random variables with

$$P(X(k)=i)=\binom{km-(m-1)i+n}{i} r^i Z^{-1}$$

  for 0 ≤ i ≤ k and P(X(k) = 0) otherwise, where Z = Z(m,n,r) is a normalizing constant. Then as $k \to \infty$

$$\frac{E[X(k)]}{k} \to \frac{m r y^{m-1}}{1+m r y^{m-1}}$$

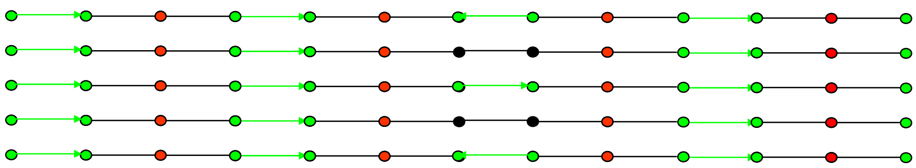
  where y is the positive real root of $1 - y - r y^m = 0$.
- Here
  - X(k) = level of the Markov chain, with $k = \lfloor (L + l - 1)/l \rfloor$
  - P(X(k) = i) = Proba(level i of the chain) = Prob(i active links) = $N(i)\pi(i)$
  - $r = \lambda/\mu$
  - m = 2l
  - n = 2((L + l - 1) mod l) + 1.
  - $E[X(k)]/k = (\sum_i i\, N(i)\, \pi(i)) / (L/l)$ for k and thus L large = $l\,\sigma$

Full capture model: spatial reuse

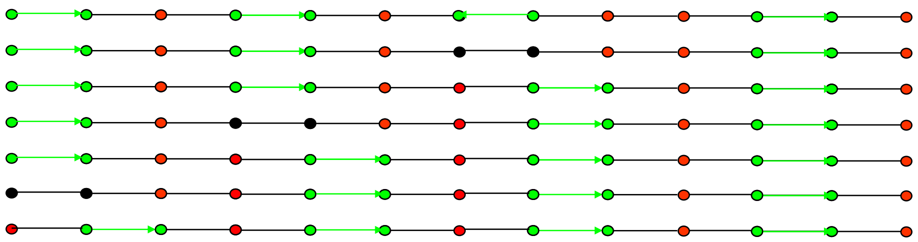
- Theorem: Symmetric exclusion domain:
  On the infinite line network
  $$\sigma = (l + \mu/(2\lambda y^{l-1}))^{-1}$$
  where y is the positive real root of $1 - y - 2\,(\lambda/\mu)\, y^{l} = 0$.
- Theorem: Asymmetric exclusion domain with full capture:
  On the infinite line network
  $$\sigma = (l + \mu/(2\lambda y^{2l-1}))^{-1}$$
  where y is the positive real root of $1 - y - (\lambda/\mu)\, y^{2l} = 0$.

Full capture model: spatial reuse

Symmetric exclusion domain



Asymmetric exclusion domain with full capture

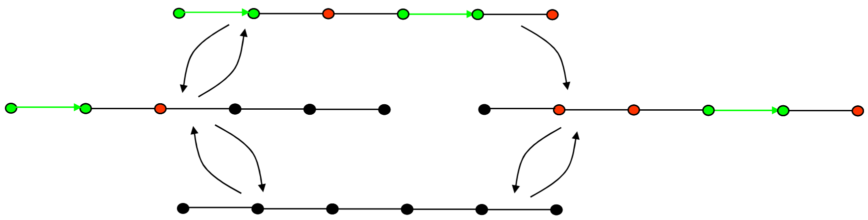


Limited Capture

- Symmetric Exclusion Domain:
  - Can consider non directed links.

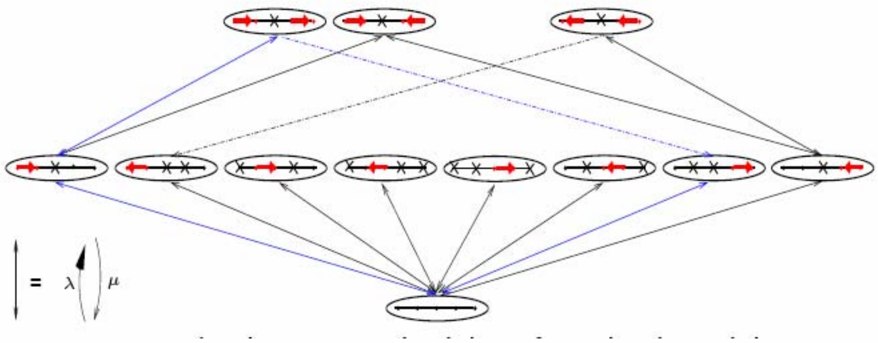
- Asymmetric Exclusion Domain and Full Capture
  - Must take directed links
  - New connection acceptance indep. of order of arrival of neighboring connections

- Asymmetric Exclusion Domain and Limited Capture
  - Must take directed links
  - New connection acceptance depends of order of arrival of neighboring connections



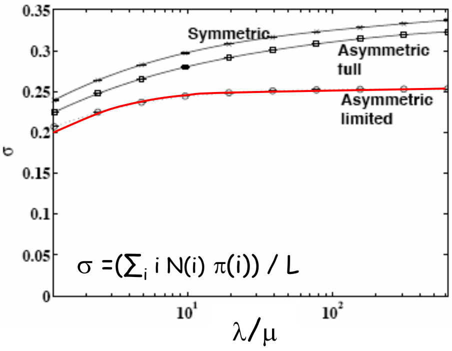
Asymmetric Excl. Dom. with Limited Capture

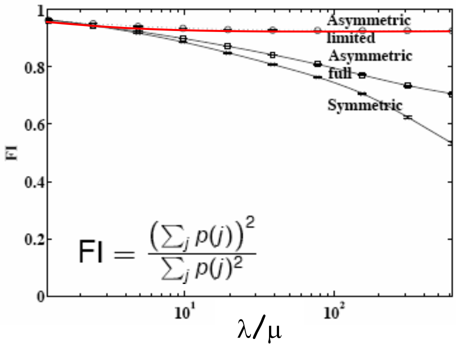
- NON Reversible Markov Chain



Limited capture model: spatial reuse

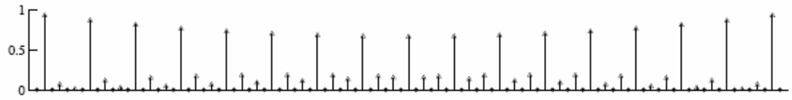
- Theorem: Let $\lambda/\mu \to \infty$
  - Symmetric exclusion domain: only the states of maximal spatial reuse have a non zero stationary probability. Hence $\sigma = 1/l$.
  - Asymmetric exclusion domain with full capture: only the states of maximal spatial reuse have a non zero stationary probability. Hence $\sigma = 1/l$.
  - Asymmetric exclusion domain with limited capture: there exist states of non maximal spatial reuse with non zero stationary probability. Hence $\sigma < 1/l$.
- What is lost in terms of spatial reuse is gained in terms of fairness.



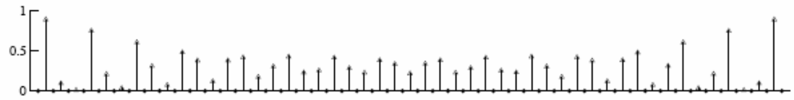


Fairness vs spatial reuse

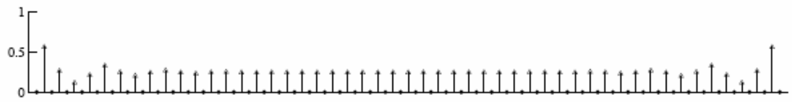
$\lambda/\mu = 600$ **Line 50 nodes**



symmetric exclusion domain, $\sigma = 0.34$, $FI = 0.53$



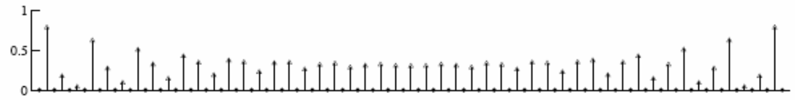
asymmetric exclusion domain, full capture, $\sigma = 0.32$, $FI = 0.70$



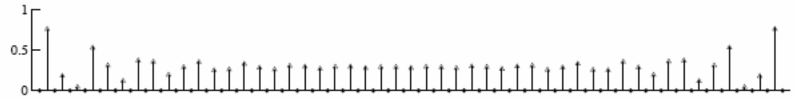
asymmetric exclusion domain, limited capture,
$\sigma = 0.25$, $FI = 0.93$

Fairness vs spatial reuse

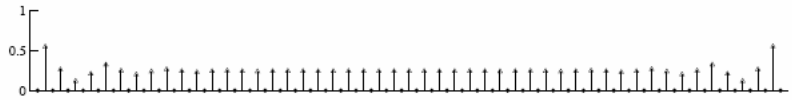
λ/μ = 20 **Line 50 nodes**



symmetric exclusion domain, $\sigma = 0.31$, $FI = 0.85$



asymmetric exclusion domain, full capture, $\sigma = 0.29$, $FI = 0.87$

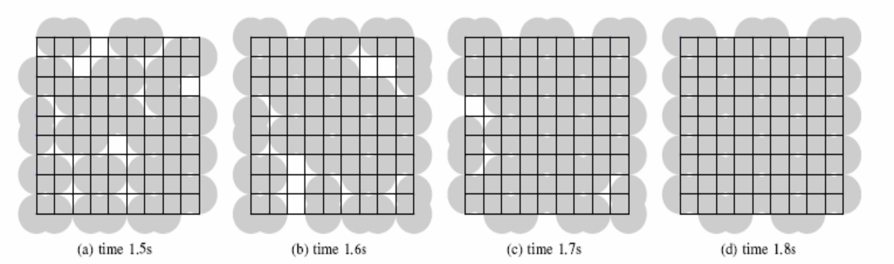


asymmetric exclusion domain, limited capture, $\sigma = 0.25$, $FI = 0.93$

Spatial reuse vs fairness

- Starvation is the price to pay for fairness (even without collisions) in 802.11 like protocols.
- Trade-off between spatial reuse and fairness can be adapted by playing on ratio λ/μ (average exchange time/average back-off time)
- Asymmetric exclusion domains (CSRange >> RXRange) lower spatial reuse but increase fairness
- Two capture models: full vs limited.
- Limited capture eliminates starvation at the cost of lower spatial reuse, full does not.
- In 1-dim network, the Markov chain is
  - reversible for symmetric and asymmetric exclusion domains, with full capture. Can compute explicitely the spatial reuse.
  - non reversible for asymmetric exclusion domains, with limited capture. Improves rotation between patterns of high spatial reuse.
- Irregularity helps!

Impact of topology



- Short term fairness: even if the network is long term fair, it takes a long time to alternate between two patterns of maximal spatial reuse.
- In 1-dim, we could compute explicitely the unique stationary distribution of the number of active links for the reversible chains.
- Similar self-organization effect in 2-dim networks + phase transition.

Scaling laws for wireless networks

- What limits can we reach? Scaling laws of wireless multi-hop networks.
- How can we coordinate nodes to reach these limits ? Self-organizing Medium Access Control (MAC) algorithms for large wireless multi-hop networks.
- Statistical physics is very helpful to solve these questions.