## Tutorial 1

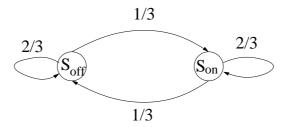
## Topic

In this tutorial, we review some basic concepts of probability theory. These concepts are very important. For example, we will use them when we study the Ethernet protocol or wireless local area networks.

Question 1: Consider a communication link which connects A with B, and suppose that A sends to B a file with N bits. During the transmission, each bit is being corrupted independently with a bit error probability p.

- (a) What is the probability  $P_{tr}$  that the file is received with out an error at B, i.e. no bit is getting corrupted during the transmission?
- (b) Suppose that B accepts the file when there is no bit error, and otherwise asks A to resend the file. Assume that B can detect perfectly whether a bit got corrupted during the transmission. What is the probability  $P_k$  that A has to send the file k times to get it accepted by B?
- (c) What is the expected number of times that A has to send the file to get it accepted at B?

Question 2: Consider a PDA (Personal Digital Assistant) which can send and receive data over a wireless communication link (for example a Wave-LAN). The PDA can be in one out of two possible states: state  $S_{off}$  (when it is not sending/receiving data) and  $S_{on}$ (when it is sending/receiving data). Assume that the PDA changes states at discrete time steps  $k = 0, 1, 2, 3, \ldots$ . When at time k the state of the PDA is equal to  $S_{off}$ , then at time (k + 1) it will be in the state  $S_{off}$  with probability 2/3 and be in the state  $S_{on}$  with probability 1/3. Similarly, when at time k in the state  $S_{on}$ , then at time (k + 1) it is in state  $S_{off}$  with probability 1/3 and in state  $S_{on}$  with probability 2/3. Assume that at time k = 0 the PDA is in state  $S_{off}$ .



- (a) Assume that at time k = 0 the PDA is in state  $S_{off}$ . What is the probability that the PDA is still in state  $S_{off}$  at time k = 1?
- (b) Assume that at time k = 0 the PDA is in state  $S_{off}$ . Using (a), what is the probability that the PDA is in state  $S_{on}$  at time k = 2, 3, 4?
- (c) Assume that the PDA is at time k = 0 in state  $S_{off}$  with probability  $P_{off}$  and in state  $S_{on}$  with probability  $P_{on}$ . Express the probability that the PDA is at time k = 1 in state  $S_{off}$ , and state  $S_{on}$ , as a function of  $P_{off}$  and  $P_{on}$ .
- (d) Find initial probabilities  $P_{off}$ ,  $P_{on}$  such that the PDA is at time k = 1 in state  $S_{off}$ , and  $S_{on}$ , again with probability  $P_{off}$ , and  $P_{on}$ , respectively. What is the interpretation of the probabilities  $P_{off}$  and  $P_{on}$ ?
- (e) Assume that at time k = 0 the PDA is in state  $S_{off}$ . Using the result of (d), try to guess that probabilities that the buffer is in state  $S_{on}$  and  $S_{off}$  after a very long time, i.e. as k approaches infinity.