$M/M/2$ queue

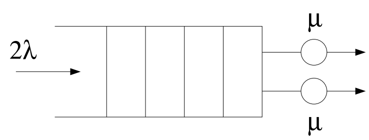

Figure 1: $M/M/2$ queue

We have the following state-transition diagram:

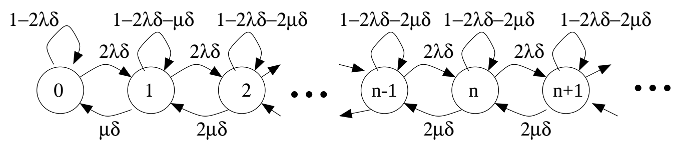

Figure 2: State diagram for $M/M/2$ queue

We have the following equations for the steady-state probabilities p_n. Setting $\rho = \frac{2\lambda}{2\mu} = \frac{\lambda}{\mu}$, we have

$$p_0 2\lambda\delta = p_1 \mu\delta,$$

or

$$p_1 = \frac{2\lambda}{\mu} = 2\rho p_0,$$

and for $n = 1, 2, ...$,

$$p_n 2\lambda\delta = p_{n+1} 2\mu\delta,$$

or

$$p_{n+1} = \frac{2\lambda}{2\mu} p_n = \frac{\lambda}{\mu} p_n = \rho p_n.$$

It follows that

$$p_n = 2\rho^n p_0, \qquad n = 1, 2, ...$$

Using the condition that

$$\sum_{n=0}^{\infty} p_n = 1,$$

we obtain that

$$\begin{aligned} \sum_{n=0}^{\infty} p_n &= p_0 + \sum_{n=1}^{\infty} 2\rho^n p_0 \\ &= p_0 + 2p_0\rho \sum_{n=1}^{\infty} \rho^{n-1} \\ &= p_0 + 2p_0\rho \sum_{n=0}^{\infty} \rho^n \\ &= p_0 + 2p_0\rho \frac{1}{1-\rho} \\ &= p_0 \left(1 + \frac{2\rho}{1-\rho}\right) \end{aligned}$$

$$
\begin{aligned}
&= p_0 \frac{1-\rho+2\rho}{1-\rho} \\
&= p_0 \frac{1+\rho}{1-\rho} \\
&= 1.
\end{aligned}
$$

It follows that

$$
\begin{aligned}
p_0 &= \frac{1-\rho}{1+\rho} \\
p_n &= 2\frac{1-\rho}{1+\rho}\rho^n = \frac{2}{1+\rho}(1-\rho)\rho^n, \qquad n = 1, 2, \ldots.
\end{aligned}
$$

Finally, we obtain that

$$
\begin{aligned}
N &= \frac{2}{1+\rho} \cdot \frac{\rho}{1-\rho} \\
T &= \frac{1}{1+\rho} \cdot \frac{1}{\mu-\lambda}
\end{aligned}
$$