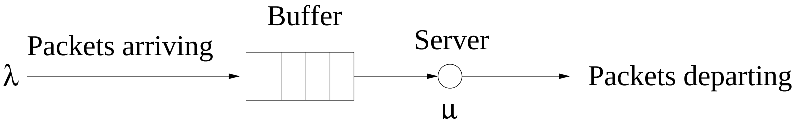
Queueing Delay



- λ: Packet arrival rate (packets/time)
- μ: Service rate (packets/time)
- $\rho = \lambda/\mu$: traffic intensity

We want to know:

- Average Number of Packets in the System
- Average Delay of a Packet (Queueing + Transmission Delay)

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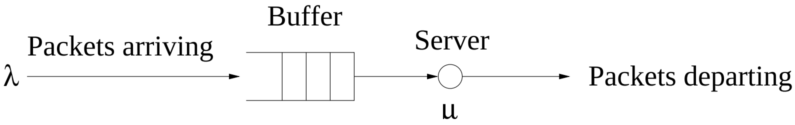

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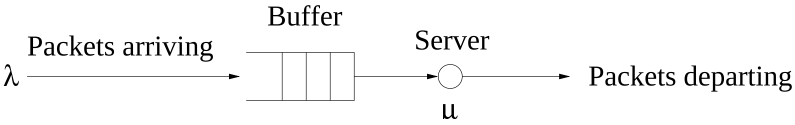

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Notation

- $A(t)$: number of packets that arrived in $[0, t]$
- $B(t)$: number of packets that departed in $[0, t]$
- $N(t) = A(t) - B(t)$: number of packets in the system (in queue and in service) at time t.
- T_i**:** Time spent in the system by the ith arriving packet.
- $N_t = \frac{1}{t} \int_0^t N(\tau) d\tau$: time average number of packets in the system up to time t.
- $N = \lim_{t \to \infty} N_t$: (time) average number of packets in the system.

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Notation (continued)

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- $\lambda = \lim_{t \to \infty} \lambda_t$: steady state arrival rate
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Outline:

- **Little's Theorem**: $N = \lambda T$
- **M/M/1 queue:** $N = \sum_{n=0}^{\infty} np_n$,
  where p_n is the steady-state probability that n packets are in the system.
- Traffic on a rainy day
- Fast-food restaurants

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Probabilistic Formulation of Little's Theorem:

So far: time average - "I observe the system for a long, long time".
Next: ensemble average - "I come back at time t to check on the system "

- $p_n(t)$: Probability that n packets are in the system at time t.
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Ergodic System and Little's Theorem:

Ergodic Systems:

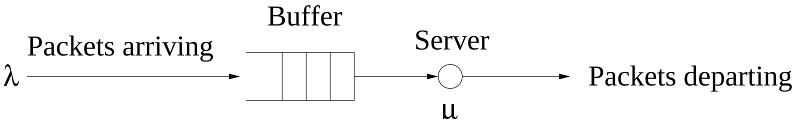
$$N = \bar{N} \qquad \text{and} \qquad T = \bar{T}$$

For ergodic systems, **Little's formula** holds with $N = \bar{N}$ and $T = \bar{T}$,

and with

$$\lambda = \lim_{t \to \infty} \frac{\text{Expected number of arrivals in the interval } [0, t]}{t}$$

M/M/1 Queue



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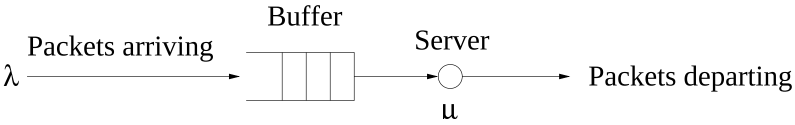

- Customers (packets) arrive according to a Poisson process
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Goal: want to determine the steady-state probability p_n that n customers are in the system.

Many Applications

- Call Centers
- Traffic planning
- Requests at a Web server