Faculty of Arts and Science
University of Toronto
CSC 358 - Introduction to Computer Networks

Tutorial 5

Topic

Queueing Theory

Question 1

Consider a $M/M/1$ queue, with a packet arrival rate λ and service rate μ such that $\rho = \frac{\lambda}{\mu} < 1$.

(a) Assume that a new packet joins the queue and finds already n packets in the system (either waiting in the queue or in service). As a function of n, $n = 0, 1, 2, ...$, what is the expected waiting time of the new packet?

(b) For a $M/M/1$, the steady-state probability that n packets are in the system is equal to

$$p_n = (1 - \rho)\rho^n, \qquad n = 0, 1, 2, ..., .$$

Using the above equation and the results of (a), compute the expected waiting time of a packet.

Question 2

Consider the $M/M/1$ that we discussed in class and let p_n, $n = 0, 1, 2, ...$, be the steady-state probability that n packets are in the system. Furthermore, let A be the event that less than n_0 packets are in the system, and let B be the event that the number of packets in the system is equal, or larger, than n_0.

(a) What is the probability of event A in steady-state, i.e. express $P\{A\}$ in terms of p_n, $n = 0, 1, 2, ...$

(b) What is the probability of event B in steady-state, i.e. express $P\{B\}$ in terms of p_n, $n = 0, 1, 2, ...$

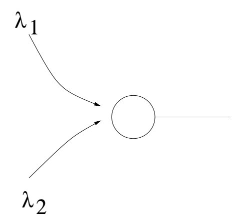
(c) Let $p_n(t)$ be the probability that n packets are in the system at time t, and let $A(t)$ be the event that at time t less than n_0 packets are in the system. For a very small δ, compute the probability $P\{A(t + \delta)\}$ as a function of $p_n(t)$, $n = 0, 1, 2, ...$, and $P\{A(t)\}$.

(d) Derive a condition on the probabilities $p_n(t)$, $n = 0, 1, 2, ...,$ which implies that $P\{A(t+\delta)\} = P\{A(t)\}$

(e) How can we use the result of (d) to derive the steady-state probabilities p_n, $n = 0, 1, 2, ...$?

Question 3

Consider a transmission system (queue) that can hold at most one packet (the packet that is in service), *i.e.* there is no buffer and a new packet either goes directly into service or is dropped.
The system receives Poisson packet traffic from two other nodes, 1 and 2, at rates λ_1 and λ_2, respectively. The service times of the packets are independently, exponentially distributed with a mean $\frac{1}{\mu}$ for packets from node 1, and $\frac{1}{2\mu}$ for packets from source 2.



(a) What is the probability that a packet that gets accepted into service is a packet from node 1?

(b) Compute the steady-state probabilities P_1, and P_2, that the system serves a packet from node 1, and node 2, respectively.