Faculty of Arts and Science

University of Toronto

CSC 358 - Introduction to Computer Networks

Tutorial 2

Topic

In the course, we will use probabilistic models for packet lengths and packet arrivals. The goal of this tutorial is to get familiar with these models.

Question 1: Packet Length

The length of data packets can vary in a wide range (some packets are very short and some packets are very long). To capture this, we model packet lengths as a random variable with a geometric distribution. That is, the probability that a packet is L bits long is given by.

$$P(L = l) = \mu(1 - \mu)^{l-1}, \qquad l \geq 1.$$

(a) Derive the average packet length, i.e. derive $E[L]$.

(b) Consider a specific packet. Assume that we know that the length of this packet is larger than l_0. Find the probability that the packet is l bits long, $l > l_0$.

(c) Derive the expected packet length when we know that $L > l_0$, i.e. derive $E[L \mid L > l_0]$.

Question 2: Packet Arrivals

In the course, we will use the following discrete-time model to characterize packet arrivals.

Suppose that time is divided into slots of length Δ_t and consider the following packet arrival process with rate λ:

1. the probability of one packet arriving during a time-slot is equal to $\lambda\Delta_t$.
2. the probability of zero arrival in the interval Δ_t is $1 - \lambda\Delta_t$.
3. arrivals are memoryless: An arrival (event) in one time interval of length Δ_t is independent of events in previous intervals.

Using this model, answer the following questions.

(a) Consider a time interval of length $k\Delta_t$. What is the probability that we have n arrivals in the time interval $[0, k\Delta_t]$ for $n = 0, ..., k$?

(b) What is the distribution of the time between two successive packet arrivals (inter-arrival time)?

Question 3: Poisson and Exponential Distribution

In this question, we show what happens for the model of Question 2 as we make Δ_t smaller and smaller, letting it approach 0.

Consider a time interval of fixed length T which is divided into N slots of equal length $\Delta_t = T/N$. In each time slot, exactly one new packet arrives with probability $\lambda\Delta_t$, and no packet arrives with probability $1 - \lambda\Delta_t$. The probability that two more packets arrive is equal to 0.

(a) What is the probability P_n that n, $n = 0, 1, ..., N$, packets arrive in the time interval $[0, T]$.

(b) Find the probability P_n as the number of time slots N approaches infinity ($N \to \infty$) (and the interval Δ_t approaches 0, $\Delta_t \to 0$). Hint: Use $\lim_{x \to 0}(1 + ax)^{\frac{k}{x}} = e^{ak}$ and for N very large, $N! \approx \frac{(N/e)^N}{\sqrt{2N\pi}}$ (Stirling's approximation).

(c) Assuming that $\Delta_t \to 0$, what is the distribution of the time between two successive packet arrivals?