Faculty of Arts and Science
University of Toronto
CSC 358 - Introduction to Computer Networks

Solutions for Tutorial 6

Topic

Unslotted Aloha

Question 1:
In unslotted Aloha (the precursor of slotted Aloha), each node (host), upon receiving a new packet, transmits the packet immediately rather than waiting for a slot boundary (slots play no role in unlotted Aloha). If the transmission times of two packets overlap at all, both packets will be lost (collison). If a packet is involved in a collision, it is retransmitted after a random delay. Assume that all packets have the same length L and the tranmission rate of the shared link is R. We rescale time and use $T = L/R$ as the new time unit.
Assume that there is an infinite number of nodes (i.e. each node has at most one packet to transmit) and let n be the number of backlogged packets at a given time. The waiting time τ until a backlogged packet is retransmitted is exponentially distributed with probability density $xe^{-\tau x}$, where the parameter x is the retranmission attempt rate. The aggregated arrival rate (over all nodes) of new packets to the system form a Poisson process with rate λ packet per unit time.

(a) Given that there n backlogged packets at time t_0 and the system is idle at time t_0, what is the probabilith that there is no transmission attempt (by a new packets or a backlogged packet) in the interval $[t_0, t_0 + t)$, $t > 0$?

The answer is $e^{-tG(n)}$, where

$$G(n) = \lambda + nx.$$

Note that the probability that no new packets arrives in the interval $[t_0, t_0 + t]$, is

$$e^{-t\lambda},$$

and the probability that none of the n backlogged packets is retransmitted is

$$e^{-tnx}.$$

(b) Let t_k be the time of the kth transmission attempt (by a new packet or a backlogged packet). For simplicity, assume that both at time t_k and t_{k+1}, there are n backlogged packets. What is the probability that the $k+1$th tranmission attempt is successful?

To have that the $k+1$th tranmission attempt is successful, we need that the $k+1$th packet does not collide with the kth and the $k+2$th packet. The $k+1$th packet does not collide with the $k+2$th packet when the time between t_{k+1} and t_{k+2} is larger than 1 unit time (the time that it takes to transmit the packet). By (a), the probability that the time between t_{k+1} and t_{k+2} is larger than 1 unit time is equal to

$$e^{-G(n)}.$$

Similarly, the probability that the $k+1$th packet does not collide with the kth packet is equal to

$$e^{-G(n)}.$$

Finally, the probability that the $k+1$ tranmission attempt is successful, is equal to the probability that $k+1$th packet does not collide with the kth and the $k+2$th packet, which is equal to

$$e^{-G(n)}e^{-G(n)} = e^{-2G(n)}.$$

(c) Assume that there are n backlogged packets, what is the throughput, i.e. the rate (of packets per unit time) of successful tranmistted packets?

As the time between retranmissions of backlogged packets is exponentially distributed, the transmission attempts (by new and backlogged packets) form a Poisson process with rate

$$G(n) = \lambda + nx.$$

By (b), each attempt is succesful with probability

$$e^{-2G(n)}.$$

Therefore, the troughput is equal to

$$G(n)e^{-2G(n)}.$$

(d) What is the maximal throughput that we can achieve?

The maximal throughtput (over all possible λ and n) is given by

$$\max_{G \geq 0} Ge^{-2G}.$$

We have

$$\frac{d}{dG}Ge^{-2G} = e^{-2G} - 2Ge^{-2G} = e^{-2G}(1-2G),$$

and we find that we obtain maximal throughput for $G = 1/2$, which is equal to

$$\frac{1}{2}e^{-1}.$$

(e) What is the optimal choice for the retransmission attempt rate x?

We achieve the maximal rate for reducing the number of backlogged packets for

$$G(n) = \lambda + nx = \frac{1}{2},$$

and it follows that the optimal x is given by

$$x = \frac{\frac{1}{2} - \lambda}{n}.$$

(f) In (e), you saw that the choice for the retransmission attempt rate x depends on n and λ. B ut a node in unslotted Aloha will not know n and λ. Can you come up with a simple heuristic for how to dynamically change x to emulate the optimal choice for the retransmission attempt rate?

There are many possbile ways of formulating a heuristic. One simple heuristic should is the following.

- whenever there is a collision, reduce x.
- whenever there is a successful transmission, increase x.