Faculty of Arts and Science
University of Toronto
CSC 358 - Introduction to Computer Networks

Solutions for Tutorial 1

Topic

In this tutorial, we review some basic concepts of probability theory. These concepts are very important. For example, we will use them when we study Ethernet and Wave-LAN.

Question 1: Consider a communication link which connects A with B, and suppose that A sends to B a file with N bits. During the transmission, each bit is being corrupted independently with a bit error probability p.

(a) What is the probability P_{tr} that the file is received with out an error at B, i.e. no bit is getting corrupted during the transmission?

$$P_{tr} = (1-p)^N$$

(b) Suppose that B accepts the file when there is no bit error, and otherwise asks A to resend the file. Assume that B can detect perfectly whether a bit got corrupted during the transmission. What is the probability P_k that A has to send the file k times to get it accepted by B?

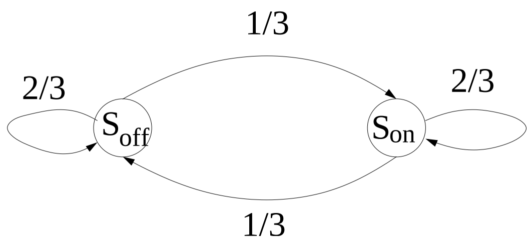
$$P_k = (1-P_{tr})^{k-1} P_{tr}$$

(c) What is the expected number of times that A has to send the file to get it accepted at B?

Let random variable T represent the number of times that A has to send the file to get it accepted at B. We have,

$$E[T] = \sum_{k=1}^{\infty} k P_k = \sum_{k=1}^{\infty} k(1-P_{tr})^{k-1} P_{tr} = \frac{1}{P_{tr}}$$

Question 2: Consider a PDA (Personal Digital Assistant) which can send and receive data over a wireless communication link (for example a Wave-LAN). The PDA can be in one out of two possible states: state S_{off} (when it is not sending/receiving data) and S_{on} (when it is sending/receiving data). Assume that the PDA changes states at discrete time steps $k = 0, 1, 2, 3, ...$ When at time k the state of the PDA is equal to S_{off}, then at time $(k+1)$ it will be in the state S_{off} with probability 2/3 and be in the state S_{on} with probability 1/3. Similarly, when at time k in the state S_{on}, then at time $(k+1)$ it is in state S_{off} with probability 1/3 and in state S_{on} with probability 2/3. Assume that at time $k = 0$ the PDA is in state S_{off}.



(a) Assume that at time $k = 0$ the PDA is in state S_{off}. What is the probability that the PDA is still in state S_{off} at time $k = 1$?

Let $P_{off}^{(1)}$ be the probability that at time $k = 1$ the PDA is in state S_{off}. We have,

$$P_{off}^{(1)} = \frac{2}{3}.$$

(b) Assume that at time $k = 0$ the PDA is in state S_{off}. Using (a), what is the probability that the PDA is in state S_{on} at time $k = 2, 3, 4$?

Let $P_{off}^{(k)}$ ($P_{on}^{(k)}$) be the probability that the PDA is in state S_{off} (S_{on}) at time k for $k = 0, 1, 2, ...$ From the initial conditions, we have that

$$P_{off}^{(0)} = 1 \qquad \text{and} \qquad P_{on}^{(0)} = 0.$$

For $k = 1, 2, 3, ...$, we have that

$$P_{off}^{(k)} = \frac{2}{3}P_{off}^{(k-1)} + \frac{1}{3}P_{on}^{(k-1)},$$

and

$$P_{on}^{(k)} = \frac{1}{3}P_{off}^{(k-1)} + \frac{2}{3}P_{on}^{(k-1)}.$$

We then obtain

$$\begin{aligned} P_{off}^{(1)} &= \tfrac{2}{3}, & P_{on}^{(1)} &= \tfrac{1}{3} \\ P_{off}^{(2)} &= \tfrac{2}{3}\tfrac{2}{3} + \tfrac{1}{3}\tfrac{1}{3} = \tfrac{5}{9}, & P_{on}^{(2)} &= \tfrac{1}{3}\tfrac{2}{3} + \tfrac{2}{3}\tfrac{1}{3} = \tfrac{4}{9} \\ P_{off}^{(3)} &= \tfrac{2}{3}\tfrac{5}{9} + \tfrac{1}{3}\tfrac{4}{9} = \tfrac{14}{27}, & P_{on}^{(3)} &= \tfrac{1}{3}\tfrac{5}{9} + \tfrac{2}{3}\tfrac{4}{9} = \tfrac{13}{27} \\ P_{off}^{(4)} &= \tfrac{2}{3}\tfrac{14}{27} + \tfrac{1}{3}\tfrac{13}{27} = \tfrac{41}{81}, & P_{on}^{(4)} &= \tfrac{1}{3}\tfrac{14}{27} + \tfrac{2}{3}\tfrac{13}{27} = \tfrac{40}{81} \end{aligned}$$

(c) Assume that the PDA is at time $k = 0$ in state S_{off} with probability P_{off} and in state S_{on} with probability P_{on}. Express the probability that the PDA is at time $k = 1$ in state S_{off}, and state S_{on}, as a function of P_{off} and P_{on}.

See question (b).

(d) Find initial probabilities P_{off}, P_{on} such that the PDA is at time $k = 1$ in state S_{off}, and S_{on}, again with probability P_{off}, and P_{on}, respectively. What is the interpretation of the probabilities P_{off} and P_{on}?

From (b) and (c), we need that

$$P_{off} = \frac{2}{3}P_{off} + \frac{1}{3}P_{on},$$

and

$$P_{on} = \frac{1}{3}P_{off} + \frac{2}{3}P_{on}.$$

Any solution for this system of equations has the property that

$$P_{off} = P_{on}.$$

As P_{off} and P_{on} are probabilities, we need that

$$P_{off} + P_{on} = 1.$$

Combining the two conditions above, we obtain that

$$P_{off} = P_{on} = \frac{1}{2}.$$

(e) Assume that at time $k = 0$ the PDA is in state S_{off}. Using the result of (d), try to guess that probabilities that the buffer is in state S_{on} and S_{off} after a very long time, i.e. as k approaches infinity.

We have

$$\lim_{k\to\infty} P_{off}^{(k)} = \frac{1}{2}$$

and

$$\lim_{k\to\infty} P_{on}^{(k)} = \frac{1}{2}$$