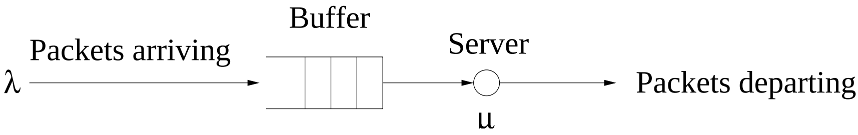
$M/M/1$ Queue



Notation

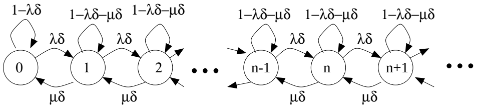
- $A(t)$: number of packets that arrived in $[0, t]$
- $B(t)$: number of packets that departed in $[0, t]$
- $N(t) = A(t) - B(t)$: number of packets in the system (in queue and in service) at time t.

Time interval $[t, t + \delta]$

- $P\{A(t + \delta) - A(t) = 1\} =$
- $P\{A(t + \delta) - A(t) = 0\} =$
- $P\{B(t + \delta) - B(t) = 1\} =$
- $P\{B(t + \delta) - B(t) = 0\} =$

For $n = 1, 2, ...$

$$\begin{aligned} P\{N(t+\delta) = n+1 \mid N(t) = n\} &= \lambda\delta \\ P\{N(t+\delta) = n-1 \mid N(t) = n\} &= \mu\delta \\ P\{N(t+\delta) = n \mid N(t) = n\} &= 1 - \lambda\delta - \mu\delta \end{aligned}$$



$$p_n = (1-\rho)\rho^n, \qquad n = 0, 1, 2, ...$$

for

$$\rho = \frac{\lambda}{\mu} < 1.$$

Next

- $N = \sum_{n=0}^{\infty} np_n = \frac{\rho}{1-\rho}$
- $T = \frac{1}{\mu-\lambda}$
- $P_{loss} \approx (1-\rho)\rho^B$
- As Internet changes - how does QoS change?
- Compare Packet-Switching with Circuit-Switching
- How should I upgrade my Network?

Average Number of Packets in the System

$$
\begin{aligned}
N &= \sum_{n=0}^{\infty} n p_n = \sum_{n=0}^{\infty} n(1-\rho)\rho^n \\
&= (1-\rho)\sum_{n=0}^{\infty} n\rho^n = (1-\rho)\rho\sum_{n=0}^{\infty} n\rho^{n-1} \\
&= (1-\rho)\rho\frac{\partial}{\partial\rho}\left(\sum_{n=0}^{\infty}\rho^n\right) \\
&= (1-\rho)\rho\frac{\partial}{\partial\rho}\left(\frac{1}{1-\rho})\right) = (1-\rho)\rho\frac{1}{(1-\rho)^2} \\
&= \frac{\rho}{1-\rho}
\end{aligned}
$$

Average Packet Delay

Using Little's Theorem, we have

$$T = \frac{1}{\lambda} N,$$

or

$$\begin{aligned} T &= \frac{1}{\lambda}\frac{\rho}{1-\rho} \\ &= \frac{1/\mu}{1-\rho} \\ &= \frac{1}{\mu-\lambda} \end{aligned}$$

Packet Loss

When the system can hold B packets (1 packet in service and $(B-1)$ packets in the buffer), then we approximate

$$P_{loss} \approx (1-\rho)\rho^B.$$

You will show this in the next assignment.