Performance Analysis

- Performance measure $R(t)$
- Example: $R(t)$ number of jobs in queue at time t

- How to characterize "system performance"?
- $R_t = \frac{1}{t}\int_0^t R(\tau)d\tau$: time average up to time t.
- $R = \lim_{t\to\infty} R_t$: time average

Performance Analysis

One Approach: time average - "I observe the system for a long, long time".
Next: ensemble average - "I come back at time t to check on the system "

- $p_n(t)$: Probability that n jobs in the system at time t.
- $\bar{R}(t) = \sum_{n=0}^{\infty} np_n(t)$**:** expected number of jobs at time t
- $\bar{R} = \lim_{t\to\infty} \bar{R}(t)$: steady-state expected number of jobs in the system

Ergodic System: $R = \bar{R}$

How to Computer R?

Assumption: $E[X] < \infty$

Result: $R = \bar{R} = \frac{E[R_n]}{E[X]}$ (under some conditions)

Approach

- Renewal Process
- Renewal Reward Processes

Renewal Process

Assumptions: $X_1, X_2, ...$ positive IID r.v.;$\bar{X} < \infty$,

Results: (Note that $0 < \bar{X}$)

$lim_{t\to\infty} \frac{N(t)}{t} = \frac{1}{\bar{X}}$

$lim_{t\to\infty} \frac{E[N(t)]}{t} = \frac{1}{\bar{X}}$

$lim_{t\to\infty} (E[N(t+\delta)] - E[N(t)]) = \frac{\delta}{\bar{X}}, \quad \delta > 0$

Intuition

Instead of arbitrary t, consider epochs S_1, S_2, ...

$N(S_n) = n$

$\lim_{n\to\infty} \frac{S_n}{n} = \bar{X}$ (Strong Law of Large Numbers)

"$\lim_{n\to\infty} \frac{N(S_n)}{S_n} = \lim_{n\to\infty} \frac{n}{S_n} = \lim_{n\to\infty} \frac{1}{S_n/n} = \frac{1}{\bar{X}}$"