Let $\pi = (\pi_1, ..., \pi_J$ be the steady state distribution of the M.C. We have

$$v(n) = r + [P]v(n-1) = r + [P]r + ... + [P]^{n-1}r + [P]^n v(0)$$

and

$$\pi v(n) = ng + \pi v(0).$$

Furthermore, we have

$$v(n) - nge = (r - ge) + [P](r - ge) + ... + [P]^{n-1}(r - ge) + [P]^n v(0)$$

and

$$\pi(v(n) - nge) = \pi v(0).$$

This leads to the conjecture that

$$\lim_{n\to\infty}(v(n) - nge) = w + \pi v(0)e,$$

where

$$w = \lim_{n\to\infty}\sum_{k=0}^{n-1}[P]^k(r - ge)$$

and

$$\pi w = 0.$$

Note that

$$\lim_{n\to\infty}(v(n) - nge) = \lim_{n\to\infty}(r + [P]v(n-1) - nge).$$

Therefore, in order to show that the conjecture is true we have to show that there exists a solution to

$$w + ge = [P]w + r$$

with

$$\pi w = 0.$$

The discussion in the text book shows that this is indeed the case.

Furthermore, the discussion in the book shows that for any solution w to

$$w + ge = [P]w + r$$

we have that

$$\lim_{n\to\infty}(v(n) - nge) = w + \beta e,$$

where

$$\beta = \pi(v(0) - w).$$