Branching Processes

- Model for the evolution of a population
- $\{X_n; n \geq 0\}$: X_n number of inviduals in generation n
- $Y_{k,n}$, $k = 1, ..., X_n$: number of offsprings of individual k in generation n
- $Y_{k,n}$: IID R.V over k and n
- $p_j = P\{Y_{k,n} = j\}$
- $X_{n+1} = \sum_{k=1}^{X_n} Y_{k,n}$
- $P_{ij} = P\left(\sum_{k=1}^{i} Y_{k,n} = j\right)$

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- **Question**: Probability that the population dies out
- **Assumption**: $p_0 > 0$,
- To answer the question, we have to study $F_{i0}(\infty)$, $i \geq 1$
- Note that $F_{i,0}(n) = \Big[F_{1,0}(n)\Big]^i$
- Therefore it suffices to focus on $F_{10}(\infty)$
- We have $F_{10}(n) = p_0 + \sum_{k=1}^{\infty} p_k \big[F_{10}(n-1)\big]^k$ and

$$F_{10}(\infty) = p_0 + \sum_{k=1}^{\infty} p_k \big[F_{10}(\infty)\big]^k$$

- We rewrite the above equation as $z = g(z) = \sum_{k=0}^{\infty} p_k z^k$