

Homework Assignment #5
Due: Tuesday, December 7, 1999, by 6:10 pm
(in your tutorial)

*On the cover page of your assignment, you must write **and sign** the following statement: "I have read and understood the policy on collaboration in homework stated in the Course Information handout." Without such a signed statement your homework will not be marked.*

1. (10 marks) Let R , S and T be arbitrary regular expressions. For each of the following assertions, state whether it is true or false, and justify your answer.

(a) If $RS \equiv RT$ and $R \neq \emptyset$ then $S \equiv T$.

(b) $(RS + R)^*R \equiv R(SR + R)^*$.

2. (20 marks) Consider the following two languages over alphabet $\Sigma = \{0, 1\}$:

$$L = \{0^n 1^m : n, m \geq 0 \text{ and } n + m \text{ is odd}\}$$

$$L' = \{x \in \{0, 1\}^* : \text{every } 0 \text{ in } x \text{ is immediately preceded and followed by } 1\}$$

For each of these languages, construct a DFSA that accepts it, and a regular expression that denotes it. Prove that your automata are correct. (You need not prove that your regular expressions are correct, but they should!)

3. (10 marks)

(a) Prove that for every language L , if L is accepted by a NFSA then it is accepted by a NFSA that has exactly one accepting state.

(b) Does the same result hold for *deterministic* FSA? Justify your answer.

4. (10 marks) Let L be a language over Σ that is accepted by some FSA. Prove that each of the following languages is also accepted by some FSA.

(a) The set of prefixes of L , $\text{Prefix}(L) = \{x \in \Sigma^* : xy \in L, \text{ for some } y \in \Sigma^*\}$.

(b) The set of maximal strings of L , $\text{Max}(L) = \{x \in L : \text{for any } y \in \Sigma^*, \text{ if } y \neq \epsilon \text{ then } xy \notin L\}$.