CSC165 Tutorial #4

Exercises

Winter 2015

Work on these exercises *before* the tutorial. You don't have to come up with a complete solution, but you should be prepared to discuss them with your TA.

1. Use the proof structure of this course to **disprove** claim S_1:

$$S_1 : \qquad \forall x \in \mathbb{R}, 2\lfloor x \rfloor > 2x - 1$$

2. Consider the definitions:

$$\forall n \in \mathbb{N}, U(n) \Leftrightarrow \exists q \in \mathbb{N}, n = 5q + 3$$
$$\forall m \in \mathbb{N}, V(m) \Leftrightarrow \exists q' \in \mathbb{N}, m = 5q' + 4$$
$$\forall p \in \mathbb{N}, W(p) \Leftrightarrow \exists q'' \in \mathbb{N}, p = 5q'' + 2$$

   Use the definitions and the proof structure of this course to prove statement S_2:

$$S_2 : \qquad \forall m \in \mathbb{N}, \forall n \in \mathbb{N}, (V(m) \wedge U(n)) \Rightarrow W(m \times n)$$

3. Use the proof structure of this course to prove the following claim:

$$S_3 : \quad \forall m \in \mathbb{N}, (\exists q_1 \in \mathbb{N}, m = 6q_1 + 2) \Rightarrow \left(\exists q_2 \in \mathbb{N}, m^2 = 6q_2 + 4\right)$$