CSC165 Tutorial #3

Sample Solutions

Winter 2015

Work on these exercises *before* the tutorial. You don't have to come up with complete solutions before the tutorial, but you should be prepared to discuss them with your TA.

Proving Equivalence

1. Prove that $P \implies (Q \implies (R \implies S))$ is equivalent to $(P \wedge Q \wedge R) \implies S$.
   Sample solution:

$$\begin{aligned} P \Rightarrow (Q \Rightarrow (R \Rightarrow S)) &\Leftrightarrow \neg P \vee (\neg Q \vee (\neg R \vee S)) && \text{[implication rule]} \\ &\Leftrightarrow (\neg P \vee \neg Q \vee \neg R) \vee S && \text{[associativity of } \vee\text{]} \\ &\Leftrightarrow \neg(P \wedge Q \wedge R) \vee S && \text{[DeMorgan's Law]} \\ &\Leftrightarrow (P \wedge Q \wedge R) \Rightarrow S && \text{[implication rule].} \end{aligned}$$

2. Prove that $((P \Rightarrow Q) \Rightarrow R) \Rightarrow S$ is equivalent to $(\neg P \wedge \neg R) \vee (Q \wedge \neg R) \vee S$.
   Sample solution:

$$\begin{aligned} ((P \Rightarrow Q) \Rightarrow R) \Rightarrow S &\Leftrightarrow \neg(\neg(\neg P \vee Q) \vee R) \vee S && \text{[implication rule]} \\ &\Leftrightarrow ((\neg P \vee Q) \wedge \neg R) \vee S && \text{[DeMorgan's Law]} \\ &\Leftrightarrow (\neg P \wedge \neg R) \vee (Q \wedge \neg R) \vee S && \text{[distributivity of } \wedge\text{]} \end{aligned}$$

Negation

1. Every dog has its day, or perhaps its cat.
   Sample solution: Some dog has neither its day nor its cat.
2. $\forall x \in X, \exists y \in Y, x > y \wedge y > x$
   Sample solution: $\exists x \in X, \forall y \in Y, x \leq y \vee y \leq x$

Guarantees

Consider the statement:

(S1) A and B are both guarantees that C is true.

1. $(A \implies C) \wedge (B \implies C)$ or $(A \vee B) \implies C$

2. “Being rich and being beautiful are both guarantees that one is hated.”

3. Suppose (S1) is true and A is false. What, if anything, can be determined about B and C? Briefly justify.
   Nothing. It tells us nothing about C, and A is unrelated to B.

4. Suppose (S1) is true and C is false. What, if anything, can be determined about A and B? Briefly justify.
   A is false and B is false. This comes from the contrapositive(s) of the implication(s), which must be true.