

CSC165 Tutorial #2

Exercises

Winter 2015

Work on these exercises *before* the tutorial. You don't have to come up with a complete solution, but you should be prepared to discuss them with your TA.

1. Consider the statements below:

$$\forall n \in \mathbb{N}, P(n) \Rightarrow Q(n)$$

$$\exists n \in \mathbb{N}, P(n) \Rightarrow Q(n)$$

$$\forall n \in \mathbb{N}, Q(n) \Rightarrow P(n)$$

$$\exists n \in \mathbb{N}, Q(n) \Rightarrow P(n)$$

Evaluate each of the four sentences above using each of the four definitions of predicates P and Q below. Briefly explain your evaluation of each of the sixteen cases.

(a) $P(n) : n < 0$ $Q(n) : n^2 < 0$

(b) $P(n) : n < 0$ $Q(n) : n^2 > 17$

(c) $P(n) : n > 3$ $Q(n) : n^2 < 0$

(d) $P(n) : n > 3$ $Q(n) : n^2 > 17$

2. Consider the statement:

S₁: For all students, missing an assignment or missing a quiz guarantees not getting a 100% in CSC165.

- (a) Write **S₁** in logical notation. The logical statement must be **well-formed**. Define all sets and predicate symbols that you use in the logical statement.
 - (b) Suppose **S₁** is true and a student didn't miss any assignments. What, if anything, can be determined about the student missing quizzes and getting 100% in CSC165? Briefly justify your answer.
 - (c) Suppose **S₁** is true and a student got 100% in CSC165. What, if anything, can be determined about the student missing assignments and quizzes? Briefly justify your answer.
3. Translate the following sentences into logical notation. The logical sentences must be **well-formed**. Define all sets and predicate symbols that you use in the logical sentences.
 - (a) If some NP-complete problem can be solved efficiently, then every NP-complete problem can be solved efficiently.
 - (b) Some courses have exactly one prerequisite course.
 - (c) Some courses have the same prerequisite courses.
 - (d) For every person, being a student is necessary for enrollment.
 4. Give the contrapositive and converse of Question 3(d) in English.