CSC165 Tutorial #1

Sample Solutions

Winter 2015

Work on these exercises *before* the tutorial. You don't have to come up with complete solutions before the tutorial, but you should be prepared to discuss them with your TA.

Here are two statements, **S1** and **S2**

S1: All pernicious humans are quixotic.

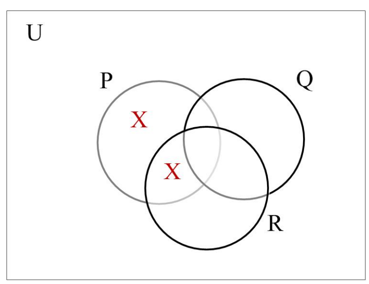
S2: Some quixotic humans are raffish.

Answer the questions below. There's no need to worry about the meanings of pernicious, quixotic, or raffish.

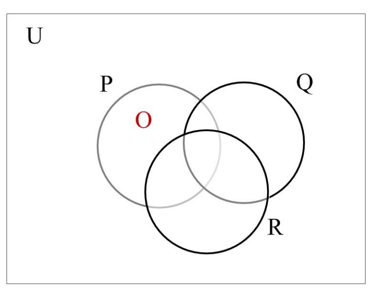
1. Draw a Venn diagram for each of the following cases. Use **X** to indicate that a region is empty and **O** to indicate that a region is **not** empty. Assume that U denotes the set of all humans. Define other sets that you need.

   Solution: Let P denote the set of pernicious humans, Q denote the set of quixotic humans, and R denote the set of raffish humans.

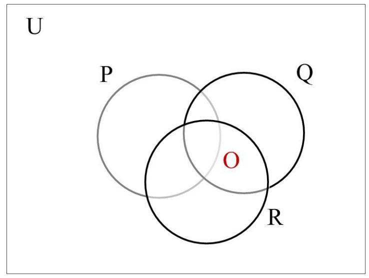
   - **S1** is true.



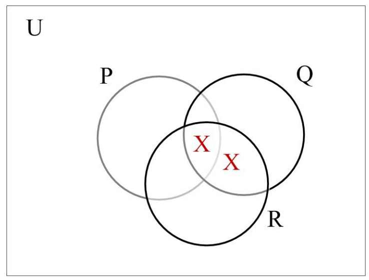
   - **S1** is false.



- **S2** is true.



- **S2** is false.



2. Suppose you can be sure that **S1** is true.
   - Does knowing that somebody is pernicious tell you whether or not they are quixotic?why?
     Yes! **S1** means that $P \subseteq Q$, so if somebody is pernicious then they are quixotic.
   - Does knowing that somebody is quixotic tell you whether or not they are pernicious?why?
     No! Consider for example an element $x \in Q \cap R$. x may or may not be in $x \in Q \cap R \cap P$.
   - Does knowing that somebody is not quixotic tell you whether or not they are not pernicious?why?
     Yes! Everything that is in P is also in Q, so if something is not in Q it cannot be in P.
   - Does knowing that somebody is not pernicious tell you whether or not they are quixotic?why?
     No! For example there might be elements in R that are not in P, but they may or my not be in Q.

3. Translate the following sentences into logical notation. Define all sets and predicate symbols that you use in the translations.
   - 0 is the smallest element of $\mathbb{N}$. ($\mathbb{N}$ denotes the set of natural numbers)
     $\forall x \in \mathbb{N}, (0 \leq x)$.
   - $\mathbb{N}$ has a smallest element.
     $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, (x \leq y)$.
   - $\mathbb{N}$ does not include a largest element.
     $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, (x < y)$.
   - Every integer number is between two integer numbers. ($\mathbb{Z}$ denotes the set of integer numbers)
     $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, \exists z \in \mathbb{Z}, (x > y) \wedge (x < z)$.
   - Everyone is loyal to someone.
     Let H denotes the set of all humans and $loyal(x, y)$ denotes that x is loyal to y.
     $\forall x \in H, \exists y \in H, loyal(x, y)$.

- All Romans were either loyal to Caesar or didn't like him.
  Let R denotes the set of Romans, $loyal(x, y)$ denotes that x is loyal to y, and $likes(x, y)$ denotes that x likes y.
  $\forall x \in R, loyal(x, Caesar) \vee \neg likes(x, Caesar)$.

4. Translate the following sentences to English.
   - $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, successor(y, x)$.
     Every natural number has a successor.
   - $\forall x \in \mathbb{N}, x = 0 \vee \exists y \in \mathbb{N}, predecessor(y, x)$.
     Every natural number is 0 or has a predecessor.
   - $\forall x \in \mathbb{N}, \neg successor(0, x)$.
     0 is not successor of any natural number.
   - $\forall x \in \mathbb{N}, \neg(x = 0) \vee \exists y \in \mathbb{N}, successor(x, y)$.
     0 is a successor of some natural number. (Note that this statement is **False**)