

PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science

Term Test #2

CSC 165H1

Section L0201

Duration — 60 minutes

No aids allowed

PLEASE HAND IN

Last Name: _____

First Name: _____

Student No: _____

Do not turn this page until you have received the signal to start.
(In the meantime, please fill out the identification section above,
and read the instructions below.)

This test consists of 3 questions on 5 pages (including this one). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Answer each question directly on the exam paper, in the space provided, and use the reverse side of the previous page for rough work. If you need more space for one of your solutions, use a blank page, **indicate** clearly the part of your work that should be marked, and indicate the **number of the page** where your answer is on the page where the question appears.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do— part marks will be given for showing that you know the general structure of an answer, even if your solutions is incomplete.

Good Luck!

IMPORTANT: For all questions, you **must** use the proof structures and format of this course. Otherwise, you won't get full mark even if your answer is correct.

Question 1. [13 MARKS]

Use the proof structure from this course to prove or disprove the following claims.

Note: You will receive **part marks** just for correctly writing the proof structure (aka the proof outline).

Part (a) [6 MARKS]

S_1 : For all integers n , $5n + 2$ is divisible by 5.

Solution: The claim is false, so I disprove it.

Here's the translation of the claim:

$$\forall n \in \mathbb{Z}, 5 \mid (5n + 2).$$

To disprove it, I must prove the negation of the claim:

$$\exists n \in \mathbb{Z}, 5 \nmid (5n + 2).$$

Let $n = 0$. Then $n \in \mathbb{Z}$. # since $0 \in \mathbb{Z}$

Then $5n + 2 = 5 \times 0 + 2 = 2$. # substitute n by 0

Then $5 \nmid 5n + 2$. # 5 does not divide 2

Then $\exists n \in \mathbb{Z}, 5 \nmid (5n + 2)$. # introduce \exists

Part (b) [7 MARKS]

S_2 : For all integers if n is odd, then $5n + 2$ is odd.

Solution: The claim is true.

Here's the translation of the claim:

$$\forall n \in \mathbb{Z}, \text{Odd}(n) \Rightarrow \text{Odd}(5n + 2).$$

Assume $n \in \mathbb{Z}$. # n is a typical integer

Assume n is odd. # antecedent

Then exists $k_0 \in \mathbb{Z}$ such that $n = 2k_0 + 1$. # definition of odd numbers

Then $5n + 2 = 10k_0 + 7$. # substitute n by $2k_0 + 1$ and algebra

Then exists $k_1 \in \mathbb{Z}$ such that $5n + 2 = 2k_1 + 1$. # $k_1 = 5k_0 + 3$, and $k_1 \in \mathbb{Z}$

Then $5n + 2$ is odd. # definition of odd numbers

Then $\text{Odd}(n) \Rightarrow \text{Odd}(5n + 2)$. # contrapositive is equivalent to the implication

Then $\forall n \in \mathbb{Z}, \text{Odd}(n) \Rightarrow \text{Odd}(5n + 2)$. # introduce \forall

Question 2. [12 MARKS]

Use the proof structure from this course to prove S_3 .

Note: You will receive **part marks** just for correctly writing the proof structure (aka the proof outline).

S_3 : For all integers n , if $n^2 + 4$ is odd, then n is odd.

Solution: Here's the translation of the claim:

$$\forall n \in \mathbb{Z}, \text{Odd}(n^2 + 4) \Rightarrow \text{Odd}(n).$$

Assume $n \in \mathbb{Z}$. # n is a typical integer

Assume n is even. # antecedent of the contrapositive

Then exists $k_0 \in \mathbb{Z}$ such that $n = 2k_0$. # definition of even numbers

Then $n^2 + 4 = (2k_0)(2k_0) + 4 = 4k_0^2 + 4$. # substitute n by $2k_0$ and algebra

Then exists $k_1 \in \mathbb{Z}$ such that $n^2 + 4 = 2k_1$. # $k_1 = 2k_0^2 + 2$, and $k_1 \in \mathbb{Z}$

Then $n^2 + 4$ is even. # definition of even numbers

Then $\text{Even}(n) \Rightarrow \text{Even}(n^2 + 4)$. # introduce \Rightarrow

Then $\text{Odd}(n^2 + 4) \Rightarrow \text{Odd}(n)$. # contrapositive is equivalent to the implication

Then $\forall n \in \mathbb{Z}, \text{Odd}(n^2 + 4) \Rightarrow \text{Odd}(n)$. # introduce \forall

Question 3. [15 MARKS]

Use **proof by contradiction** to prove that there is no integer n such that $(n \equiv 4 \pmod{5})$ and $(n \equiv 1 \pmod{10})$.

Note 1: You will receive **part marks** just for correctly writing the proof structure (aka the proof outline).

Note 2: You **must** use the proof structure from this course.

Hint: Recall that for integers x, y, z , the notation $x \equiv y \pmod{z}$ means " $x - y$ is a multiple of z ."

Solution: Here's the translation of the claim:

$$\neg(\exists n \in \mathbb{Z}, (n \equiv 4 \pmod{5}) \wedge (n \equiv 1 \pmod{10})).$$

We must assume the negation of the claim and then derive a contradiction.

Assume $\exists n \in \mathbb{Z}, (n \equiv 4 \pmod{5}) \wedge (n \equiv 1 \pmod{10})$. # to derive a contradiction

Then exists $k_0 \in \mathbb{Z}$ such that $n - 4 = 5k_0$. # definition *mod*

Then $n = 5k_0 + 4$. # add 5 to both sides of the above equality

Also exists $k_1 \in \mathbb{Z}$ such that $n - 1 = 10k_1$. # definition *mod*

Then $n = 10k_1 + 1$. # add 3 to both sides of the above equality

Then $5k_0 + 4 = 10k_1 + 1$. # both are equal to n

Then $10k_1 - 5k_0 = 3$. # subtract $5k_0 + 1$ from both sides

Then $2k_1 - k_0 = 3/5$. # divide both sides by 5

Contradiction! # $2k_1 - k_0 \in \mathbb{Z}$, but $3/5$ is not an integer

Then $\neg(\exists n \in \mathbb{Z}, (n \equiv 4 \pmod{5}) \wedge (n \equiv 1 \pmod{10}))$. # assuming the negation leads to a contradiction

This page is left (nearly) blank to accommodate work that wouldn't fit elsewhere.

1: _____/13

2: _____/12

3: _____/15

TOTAL: _____/40