PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science

Term test #1

CSC 165H1
Section L0101
Duration — 60 minutes

No aids allowed

PLEASE HAND IN

Last Name: Solutions______________________________

First Name: Sample______________________________

Student ID: ______________________________

Do **not** *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above,
and read the instructions below.)

This test consists of 4 questions on 6 pages (including this one). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Please answer questions in the space provided.

Good Luck!

Question 1. [14 MARKS]

- Express each of the following sentences in logical notation.
  Define all set and predicate symbols that you use in the logical expressions.
- Write the **negation** of each of the sentences in **English** and in **logical** form.
  Simplify the logical sentences so that only predicates are negated.

Part (a) [4 MARKS] Not everybody is your friend or someone is not perfect.
Solution :
Let P be the set of all persons, $Friend(x)$ denotes x is your friend and $Perfect(x)$ denotes x is perfect.
$\exists x \in P, \neg Friend(x) \vee \exists x \in P, \neg Perfect(x)$
Negation :
$\forall x \in P, Friend(x) \wedge \forall x \in P, Perfect(x)$
Everybody is your friend and everyone is perfect.

Part (b) [4 MARKS] BA3201 can be accessed by every student if that student is enrolled in CSC165.
Solution :
Let S be the set of all students, $enrolled(x, y)$ denotes x is enrolled in y and $access(x, y)$ denotes x has access to y.
$\forall x \in S, enrolled(x, CSC165) \Rightarrow access(x, BA3201)$
Negation :
$\exists x \in S, enrolled(x, CSC165) \wedge \neg access(x, BA3201)$
Some students who are enrolled in CSC165 does not have access to BA3201.

Part (c) [3 MARKS] Some students in CSC165 do not participate in lectures.
Solution :
Let S be the set of all students in CSC165, $Participate(x)$ denotes x participates in lectures.
$\exists x \in S, \neg Participate(x)$
Negation :
$\forall x \in S, Participate(x)$
All the students in CSC165 participate in lectures.

Part (d) [3 MARKS] At least one student in CSC165 watches TV every night.
You **must only** use the following symbols in your logical sentence:
S: the set of all students in CSC165.
N: the set of all nights.
$W(x, y)$: x watch TV on night y.

Solution :
$\exists x \in S, \forall y \in N, W(x, y)$
Negation :
$\forall x \in S, \exists y \in N, \neg W(x, y)$
For all students in CSC165 there are some nights that they do not watch TV.

Question 2. [6 MARKS]

Let $V(x, y)$ denotes x has visited y, S denotes the set of all students in CSC165, and R denotes the set of all stores.

Express each of the statements by a simple English sentence.

Avoid symbols (e.g. x) and predicates (e.g. T(x,y)) in English sentences.

Part (a) [1.5 MARK]

$V(Charles, Indigo)$
Solution : Charles visited Indigo.

Part (b) [1.5 MARK]

$\exists x \in S, V(x, Apple)$
Solution : Some CSC165 student visited Apple.

Part (c) [3 MARKS]

$\exists y \in S, \forall z \in R, (y \neq Tom) \wedge (V(Tom, z) \implies V(y, z))$
Solution : At least one CSC165 student, who is not Tom, visited any store Tom visited.

Question 3. [10 MARKS]

Verify if the following arguments are logically valid. Provide **logical justifications** for your answers.

(Hint: it might be helpful if you re-state the arguments in symbolic notation)

Part (a) [6 MARKS]

Given the following **assumptions:**
AS1: Going over the speed limit is sufficient for getting a ticket.
AS2: I did not go over the speed limit.

we can **conclude** that:
Con1: I did not get a ticket.

Solution:
P: Going over the speed limit.
Q: Getting a ticket.

AS1 can be re-stated as $P \Rightarrow Q$.
AS2 can be re-stated as $\neg P$.
Con1 can be re-stated as $\neg Q$.

The argument is **invalid**, since given $P \Rightarrow Q$ and $\neg P$, it is not logically possible to conclude $\neg Q$.

Part (b) [4 MARKS]

Given the following **assumptions:**
AS3: Whenever I am happy, I dance.
AS4: I am happy now.

we can **conclude** that:
Con2: I am dancing now.

Solution:
P: I am happy.
Q: I dance.

AS3 can be re-stated as $P \Rightarrow Q$.
AS4 can be re-stated as P.
Con2 can be re-stated as Q.

The argument is **valid**, since $P \Rightarrow Q$ and P, implies Q.

Question 4. [20 MARKS]

For each statement below, identify whether it is satisfiable, unsatisfiable, or is a tautology, and **prove** your answer.

To prove that a statement is a tautology you **must** use **manipulation rules** (justify each step of your derivation by naming the rule). Use **truth tables** to justify satisfiability or unsatisfiability.

Part (a) [11 MARKS]

$P \wedge (P \Rightarrow Q) \Rightarrow Q$

Solution: The statement is a tautology.

$$\begin{aligned}
(P \wedge (P \Rightarrow Q) \Rightarrow Q) &\Leftrightarrow (\neg(P \wedge (\neg P \vee Q)) \vee Q) && \text{(Implication Rule)}\\
&\Leftrightarrow ((\neg P \vee \neg(\neg P \vee Q)) \vee Q) && \text{(DeMorgan's law)}\\
&\Leftrightarrow ((\neg(\neg P \vee Q) \vee \neg P) \vee Q) && \text{(Commutativity)}\\
&\Leftrightarrow (\neg(\neg P \vee Q) \vee (\neg P \vee Q)) && \text{(Associativity)}\\
&\Leftrightarrow \mathit{True} && \text{(Def. of } \vee\text{)}
\end{aligned}$$

Part (b) [9 MARKS]

$(P \Rightarrow (Q \Leftrightarrow R)) \Rightarrow (Q \Leftrightarrow R)$

Solution: The statement is satisfiable.

P	Q	R	$(Q \Leftrightarrow R)$	$P \Rightarrow (Q \Leftrightarrow R)$	$(P \Rightarrow (Q \Leftrightarrow R)) \Rightarrow (Q \Leftrightarrow R)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	T	T	T

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# 2:	______/ 6
# 3:	______/10
# 4:	______/20
TOTAL:	______/50