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UNIVERSITY OF TORONTO
Faculty of Arts and Science

Term test #1

CSC 165H1
Section L0201
Duration — 60 minutes

No aids allowed

Last Name: Solutions

First Name: Sample

Student ID: _____

Do **not** *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below.)

This test consists of 4 questions on 6 pages (including this one). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Please answer questions in the space provided.

Good Luck!

Question 1. [14 MARKS]

- Express each of the following sentences in logical notation.

  Define all set and predicate symbols that you use in the logical expressions.

- Write the **negation** of each of the sentences in **English** and in **logical** form.

  Simplify the logical sentences so that only predicates are negated.

All logical sentences must be **well-formed**.

Part (a) [3 MARKS] At least one of your friends is perfect.
Solution :
Let F be the set of all your friends, and $Perfect(x)$ denotes x is perfect.
$\exists x \in F, Perfect(x)$
Negation :
$\forall x \in F, \neg Perfect(x)$
All of your friends are not perfect.

Part (b) [5 MARKS] Students graduate if they pass all courses.
Solution :
Let S be the set of all students, C be the set of all courses, $Graduate(x)$ denotes x graduated and $Pass(x, y)$ denotes x has passed y.
$\forall x \in S, (\forall y \in C, Pass(x, y)) \Rightarrow Graduate(x)$
Negation :
$\exists x \in S, \forall y \in C, Pass(x, y) \wedge \neg Graduate(x)$
Some students passed all courses but have not graduated.

Part (c) [3 MARKS] Everyone in CSC165 has studied a foreign language.
Solution :
Let S be the set of all students in CSC165, and $F(x)$ denotes x has studied a foreign language.
$\forall x \in S, F(x)$
Negation :
$\exists x \in S, \neg F(x)$
Some students in CSC165 have not studies a foreign language.

Part (d) [3 MARKS] Each person in the race participated in at least one competition.
You **must only** use the following symbols in your logical sentence:
P: the set of all persons in the race.
C: the set of all competitions.
$Par(x, y)$: x participated in competition y.

Solution :
$\forall x \in P, \exists y \in C, Par(x, y)$
Negation :

$\exists x \in P, \forall y \in C, \neg Par(x, y)$
Some persons in the race did not participate in any competitions.

Question 2. [6 MARKS]

Let $V(x, y)$ denotes x has visited y, S denotes the set of all students in CSC165, and R denotes the set of all stores.

Express each of the statements by a simple English sentence.

Avoid symbols (e.g. x) and predicates (e.g. T(x,y)) in English sentences.

Part (a) [1.5 MARK]

$\exists y \in R, V(Ella, y)$.
Solution : Ella visited some store.

Part (b) [1.5 MARK]

$\exists x \in S, \forall y \in R, V(x, y)$
Solution : Some CSC165 student visited all stores.

Part (c) [3 MARKS]

$\exists x \in S, \exists y \in S, \forall z \in R, (x \neq y) \wedge (V(x, z) \Leftrightarrow V(y, z))$
Solution : At least two different CSC165 students visited exactly the same stores.

Question 3. [10 MARKS]

Verify if the following arguments are logically valid. Provide **logical justifications** for your answers.

(Hint: it might be helpful if you re-state the arguments in symbolic notation)

Part (a) [4 MARKS]

Given the following **assumption:**
AS1: There is no alive dinosaur.

we can **conclude** that:
Con1: Alive dinosaurs eat humans.

Solution:
D: set of all dinosaurs.
$L(x)$: x is alive.
$E(x)$: x eats humans.

AS1 can be re-stated as $\neg\exists x \in D, L(x)$.

Con1 can be re-stated as $\forall x \in D, L(x) \Rightarrow E(x)$.

The argument is **valid** because **Con1** is vacuously true.

Part (b) [6 MARKS]

Given the following **assumptions:**
AS2: A decent education is necessary for getting a job.
AS3: You have a decent education.

we can **conclude** that:
Con2: You will get a job.

Solution:
P: Having a decent education.
Q: Getting a job.

AS2 can be re-stated as $Q \Rightarrow P$.
AS3 can be re-stated as P.
Con2 can be re-stated as Q.

The argument is **invalid**, since given $Q \Rightarrow P$ and P, it is not logically possible to conclude Q.

Question 4. [20 MARKS]

For each statement below, identify whether it is satisfiable, unsatisfiable, or is a tautology, and **prove** your answer.

To prove that a statement is a tautology you **must** use **manipulation rules** (justify each step of your derivation by naming the rule). Use **truth tables** to justify satisfiability or unsatisfiability.

Part (a) [9 MARKS]

$(P \Rightarrow Q) \wedge (P \Rightarrow R) \Rightarrow (Q \Leftrightarrow R)$

Solution: The statement is satisfiable.

P	Q	R	$P \Rightarrow Q$	$P \Rightarrow R$	$(P \Rightarrow Q) \wedge (P \Rightarrow R)$	$Q \Leftrightarrow R$	$(P \Rightarrow Q) \wedge (P \Rightarrow R) \Rightarrow (Q \Leftrightarrow R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	F	T
T	F	F	F	F	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

Part (b) [11 MARKS]

$(P \Rightarrow Q) \wedge ((P \wedge Q) \wedge R) \Rightarrow R$

Solution: The statement is a tautology.

$$
\begin{aligned}
((P \Rightarrow Q) \wedge ((P \wedge Q) \wedge R) \Rightarrow R) &\Leftrightarrow (\neg(\neg P \vee Q) \wedge ((P \wedge Q) \wedge R)) \vee R) && \text{(Implication Rule)} \\
&\Leftrightarrow ((P \wedge \neg Q) \vee ((\neg P \vee \neg Q) \vee \neg R)) \vee R) && \text{(DeMorgan's law)} \\
&\Leftrightarrow ((P \wedge \neg Q) \vee (\neg P \vee \neg Q)) \vee (\neg R \vee R)) && \text{(Associativity)} \\
&\Leftrightarrow ((P \wedge \neg Q) \vee (\neg P \vee \neg Q)) \vee \mathit{True}) && \text{(Def. of } \vee\text{)} \\
&\Leftrightarrow \mathit{True} && \text{(Def. of } \vee\text{)}
\end{aligned}
$$

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1: _____/14

2: _____/ 6

3: _____/10

4: _____/20

TOTAL: _____/50