CSC165
Mathematical Expression and Reasoning for Computer Science

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Department of Computer Science
University of Toronto

Winter 2015

General Info

- Instructors:
  - **Bahar Aameri**
    Email: bahar at cs.toronto.edu
    Office hours: Friday 12:30-1:30pm and 3:30-5pm in BA4261.
  - **Lisa Yan**
    Email: lyan at cs.toronto.edu
    Office hours: Wednesday 3-5 in BA4261.
- Lectures:
  - Section **L0101**: MWF **11-noon**
  - Section **L0201**: MWF **2-3 pm**

  Bahar teaches the **Monday** morning, **Friday** morning and **Friday** afternoon sessions.
  Lisa teaches the **Monday** afternoon, **Wednesday** morning and **Wednesday** afternoon sessions.

Other Info

- **Course web page**:
  www.cdf.toronto.edu/~csc165h/winter

- **Course info sheet** (**important**):
  http://www.cdf.toronto.edu/~csc165h/winter/165infosheet.pdf

- **Course Notes**:
  http://www.cdf.toronto.edu/~csc165h/winter/165notes.pdf

Tutorials

- Section **L0101**: **Tuesdays** 2:10-3:30 pm
- Section **L0201**: **Thursdays** 2:10-3:30 pm
- Format:
  - Work on some **exercises** (40 mins)
  - **Discuss** solutions with the TA (20 mins)
  - Take an easy **quiz** (10-15 mins)
- **Room assignments** for tutorials will be announced on the course web page by the end of Week 1.
- **You must take the quizzes in the room that you are assigned to.**

Tutorials start on Week 2

Course Work

Item	Due	Weight
Assignment #1	Fri Jan 30, 11:59 pm	10%
Assignment #2	Fri Mar 06, 11:59 pm	10%
Assignment #3	**Thu** April 02, 11:59 pm	10%
Quizzes	Weeks 2, 3, 4, 6, 8, 9, 11, 12	10% (in total)
Term test #1	Tue Feb 03 or Thu Feb 05	10%
Term test #2	Tue Mar 10 or Thu Mar 12	10%
Final exam	Some time in April	40%

- **Quizzes**: at the end of **tutorials** (note that Week 7 is the reading week).
- **Term Tests**: Week 5 and Week 10, during the **tutorial times**.
- **Final exam**: In order to pass the course, you must obtain **at least 40%** on the final exam.

Assignments

- Assignments may be submitted in groups of up to **two** students.
  - You may choose your group-mate from students in the other section.
  - You may change your group-mate for each assignment.
- Each group must submit a single **PDF** file on *MarkUs*
  - Submissions must be **typed** (LaTeX is strongly recommended)
  - **Recommendation**: work on all questions individually! Then discuss your answers with your group-mate and prepare the final submission together.
- **Late Submission**: **One time** 24-hour grace period with no penalty (per person).
- **Remark requests** for assignments must be submitted through *MarkUs* within **one week** of receiving the assignment back.

Email Policy and Discussion Board

- General questions about the course or assignments should be submitted to the discussion board (**Piazza**): http://piazza.com/utoronto.ca/winter2015/csc165
  - Use your **UTOR email** (mail.utoronto.ca) to sign up.
  - The discussion board will be monitored by TAs and instructors.
  - Don't discuss assignment solutions until **24h after** the due dates.
- Use email only for personal issues such as requesting special considerations.

How to do well in CSC165

- Check the course web page and emails regularly.
- Come to the lectures and participate
- Work on tutorials, show your solution to the TA, ask for feedback.
- Spend 8-10 hours/week:
  - 3 hours in lectures
  - 2 hours in tutorial
  - 3-5 hours reviewing slides and course notes, working on assignments.
- Need more practice?
  http://www.cdf.toronto.edu/~csc165h/winter/165PraQueSoln.pdf

Chapter 1

Introduction

What's CSC165 about?

- **Logic and Mathematical Expression**: how to communicate precisely using a formal language.
- **Proof Techniques**: how to make logical, convincing arguments.
- **Algorithm Analysis and Complexity**: how to prove correctness of computer programs and measure their running times.
- **Basic Introduction to Computability**: what computers cannot do.

Why do we need CSC165?!

As a **Software Developer/Designer**

- You need to **communicate** with your colleagues and customers about the software requirements
- You need to be able to **prove** that your codes **satisfies** the requirements
- You need to **solve** a **mathematical problem** in order to implement a requirement

As a **Researcher**

- **Artificial intelligence** ⇒ Mathematical Logic, Set Theory, Probability
- **Cryptography** ⇒ Number Theory, Field Theory
- **Algorithm** ⇒ Combinatorics, Set Theory
- **Programming Languages** ⇒ Mathematical Logic, Set Theory
- **Databases** ⇒ Mathematical Logic, Set Theory
- **Networking** ⇒ Graph Theory, Statistics

How to solve a problem

Understand the problem:

- Know what is **required**
- Know what is **given**
  - usually the given information is necessary for solving the problem.
  - BUT, sometimes completing a proof needs information that are not given but **have been proved** or are **well-known** to be true.
- **Re-state** the problem in your own words;
- Might help to draw some **diagrams**.

Plan solution(s):

- If you have seen something **similar**, you may be able to use its *result* or its *method*.
- Work **backwards**: assume you have solved the problem and deduce the next-to-last step.
- Try solving **simpler versions** of the problem.

How to solve a problem

Carry out your plan:

- You may need to **repeat** (parts of) the earlier steps.
- If you are still stuck, identify *exactly* what information/assumptions you require that are missing and find a way to achieve them.

Review your solution:

- Verify that your solution is correct and convincing.
- Extend the solution to new problems.

Apply the problem-solving approach to the following puzzle

Streetcar Drama

On a streetcar, you overhear the following conversation:

A: Haven't seen you in a long time! How old are your three kids now?

B: The product of their ages (in years) is 36.

A: That doesn't really answer my question.

B: Well, the sum of their ages is — [fire engine goes by]

A: Still, that doesn't tell me how old they are.

B: Well, the eldest plays piano.

A: Okay, I see: their ages are — [you have to get off the streetcar]

Problem Solving Exercise: solution

Streetcar Drama → Understand the problem

On a streetcar, you overhear the following conversation:

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A: That doesn't really answer my question.

B: Well, the sum of their ages is — [fire engine goes by]

A: Still, that doesn't tell me how old they are.

B: Well, the eldest plays piano.

A: Okay, I see: their ages are —

What is required?

Problem Solving Exercise: solution

Streetcar Drama → **Understand the problem**

On a streetcar, you overhear the following conversation:

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A: That doesn't really answer my question.

B: Well, the sum of their ages is — [fire engine goes by]

A: Still, that doesn't tell me how old they are.

B: Well, the eldest plays piano.

A: Okay, I see: their ages are —

What is required?

The kids ages!

Let's denote the kids' ages by **x**, **y** and **z**.

Problem Solving Exercise: solution

Streetcar Drama → Understand the problem

A: Haven't seen you in a long time! How old are your three kids now?

B: The product of their ages (in years) is **36**. → $\mathbf{x \cdot y \cdot z = 36}$

B: The sum of their ages **is ...**

A: Still, that doesn't tell me how old they are. → $\mathbf{x + y + z}$ **is not unique among all possible combinations**

B: Well, the **eldest** plays piano. → $\mathbf{x > y}$ **and** $\mathbf{x > z}$

A: Okay, I see: their **ages are ...** → **determine the value of** $\mathbf{x, y}$ **and** $\mathbf{z}$.

Problem Solving Exercise: solution

Streetcar Drama → **Understand the problem**

- What is given?
  1. $x \cdot y \cdot z = 36$ (**Given**)
  2. $x + y + z$ is not unique among all possible combinations (**Deduced from the given info**)
  3. $x > y$ and $x > z$ (**well-known to be true**)
- What is required?
  - The value of x, y and z.

Problem Solving Exercise: solution

Streetcar Drama → **A plan for the solution**

1. $\mathbf{x \cdot y \cdot z = 36}$,
2. $x + y + z$ is not unique among all possible combinations,
3. $x > y$ and $x > z$.

Possible Answers:

$\mathbf{36, 1, 1}$

$\mathbf{18, 2, 1}$

$\mathbf{12, 3, 1}$

$\mathbf{9, 4, 1}$

$\mathbf{9, 2, 2}$

$\mathbf{6, 6, 1}$

$\mathbf{6, 3, 2}$

$\mathbf{4, 3, 3}$

Streetcar Drama: solution

Streetcar Drama → A plan for the solution

1. $x \cdot y \cdot z = 36$,
2. $\mathbf{x + y + z}$ **is not unique among all possible combinations**,
3. $x > y$ and $x > z$.

Possible Answers:

$36, 1, 1$ where $\mathbf{36 + 1 + 1 = 38}$

$18, 2, 1$ where $\mathbf{18 + 2 + 1 = 21}$

$12, 3, 1$ where $\mathbf{12 + 3 + 1 = 16}$

$9, 4, 1$ where $\mathbf{9 + 4 + 1 = 14}$

$9, 2, 2$ where $\mathbf{9 + 2 + 2 = 13}$

$6, 6, 1$ where $\mathbf{6 + 6 + 1 = 13}$

$6, 3, 2$ where $\mathbf{6 + 3 + 2 = 11}$

$4, 3, 3$ where $\mathbf{4 + 3 + 3 = 10}$

Problem Solving Exercise: solution

Streetcar Drama → **A plan for the solution**

1. $x \cdot y \cdot z = 36$,
2. $\mathbf{x + y + z}$ **is not unique among all possible combinations**,
3. $x > y$ and $x > z$.

Possible Answers:

$36, 1, 1$ where $\mathbf{36 + 1 + 1 = 38}$

$18, 2, 1$ where $\mathbf{18 + 2 + 1 = 21}$

$12, 3, 1$ where $\mathbf{12 + 3 + 1 = 16}$

$9, 4, 1$ where $\mathbf{9 + 4 + 1 = 14}$

$\mathbf{9, 2, 2}$ where $\mathbf{9 + 2 + 2 = 13}$

$\mathbf{6, 6, 1}$ where $\mathbf{6 + 6 + 1 = 13}$

$6, 3, 2$ where $\mathbf{6 + 3 + 2 = 11}$

$4, 3, 3$ where $\mathbf{4 + 3 + 3 = 10}$

Streetcar Drama → A plan for the solution

1. $x \cdot y \cdot z = 36$,
2. $x + y + z$ is not unique among all possible combinations,
3. $\mathbf{x > y}$ **and** $\mathbf{x > z}$.

Possible Answers:

$\mathbf{9, 2, 2}$

$\mathbf{6, 6, 1}$

Problem Solving Exercise: solution

Streetcar Drama → A plan for the solution

1. $x \cdot y \cdot z = 36$,
2. $x + y + z$ is not unique among all possible combinations,
3. $\mathbf{x > y}$ **and** $\mathbf{x > z}$.

Possible Answers:

$\mathbf{9, 2, 2}$

$6, 6, 1$

Problem Solving Exercise

Apply the problem-solving approach to the following puzzle

Court Case

At a murder trial, four witnesses give the following testimony.

- Alice: If either Bob or Carol is innocent, then so am I.
- Bob: Alice is guilty, and either Carol or Dan is guilty.
- Carol: If Bob is innocent, then Dan is guilty.
- Dan: If Bob is guilty, then Carol is innocent; however, Bob is innocent.

Is the testimony consistent, i.e., is it possible that everyone is telling the truth?

Problem Solving Exercise:solution

Court Case → **Restate the problem**

First, we define the following abbreviations:
A: Alice is innocent
B: Bob is innocent
C: Carol is innocent
D: Dan is innocent
Then we restate the testimonies to the following form:

- Alice: if (**B** or **C**) then **A**
- Bob: (not **A**) and ((not **C**) or (not **D**))
- Carol: if **B** then (not **D**)
- Dan: (if (not **B**) then **C**) and **B**

Problem Solving Exercise:solution

Court Case → A plan for the solution

- Alice: if (**B** or **C**) then **A**
- Bob: (not **A**) and ((not **C**) or (not **D**))
- Carol: if **B** then (not **D**)
- Dan: (if (not **B**) then **C**) and **B**

- First, we assume that **all** testimonies are **true**.
- We start by **Bob**'s and **Dan**'s testimonies because they make unconditional accusation/vindication:
  - Bob says that Alice is not innocent (not **A**).
  - Dan says that Bob is innocent (**B**).
- Then, according to the Alice's testimony Bob and Carol are not innocent (because otherwise Alice must be innocent too). But this **contradicts** Dan's testimony.

Therefore, at least one person is **lying**.