Chapter 3

Formal Proofs

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Announcements

- **Assignment 2** is posted.
  - **Due date**: Mar 06, before midnight on MarkUs.
  - Assignments may be submitted in groups of up to **two** students. You may choose your group-mate from students in the other section.
  - Submissions must be **typed**.
  - The size of the PDF file **must** be less than **1MB**.
- **Mid-term Course Evaluation**: link posted on the course website.
  - Please fill out to let us know your comments/suggestions.
  - It is a Google form, but you **don't** have to be logged-in.
  - It is completely anonymous.
  - Takes less than 5 minutes.
- **Assignment 1** will be return during the weekend.
  - **Remark requests** through MarkUs by **Wednesday, Feb 25**.

Today's Topics

- **Exercise: Formal Proofs**

Formal Proofs

Exercises

Use the proof structures in this course to **prove** or **disprove** the following claims

1. Consider the definition of the floor function:

   $$\mathbf{D_1}: \forall x \in \mathbb{R}, \forall y \in \mathbb{Z}, (y = \lfloor x \rfloor) \Leftrightarrow (y \leq x) \wedge (\forall z \in \mathbb{Z}, (z \leq x) \Rightarrow (z \leq y)).$$

   Use $\mathbf{D_1}$ to prove $\forall x \in \mathbb{R}, (\lfloor x \rfloor > x - 1)$

2. Use **proof by contradiction** to prove that $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x > y \Rightarrow \lfloor x \rfloor \geq \lfloor y \rfloor$

3. For $x \in \mathbb{R}$,

   $$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

   Prove $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, |x||y| = |xy|$.